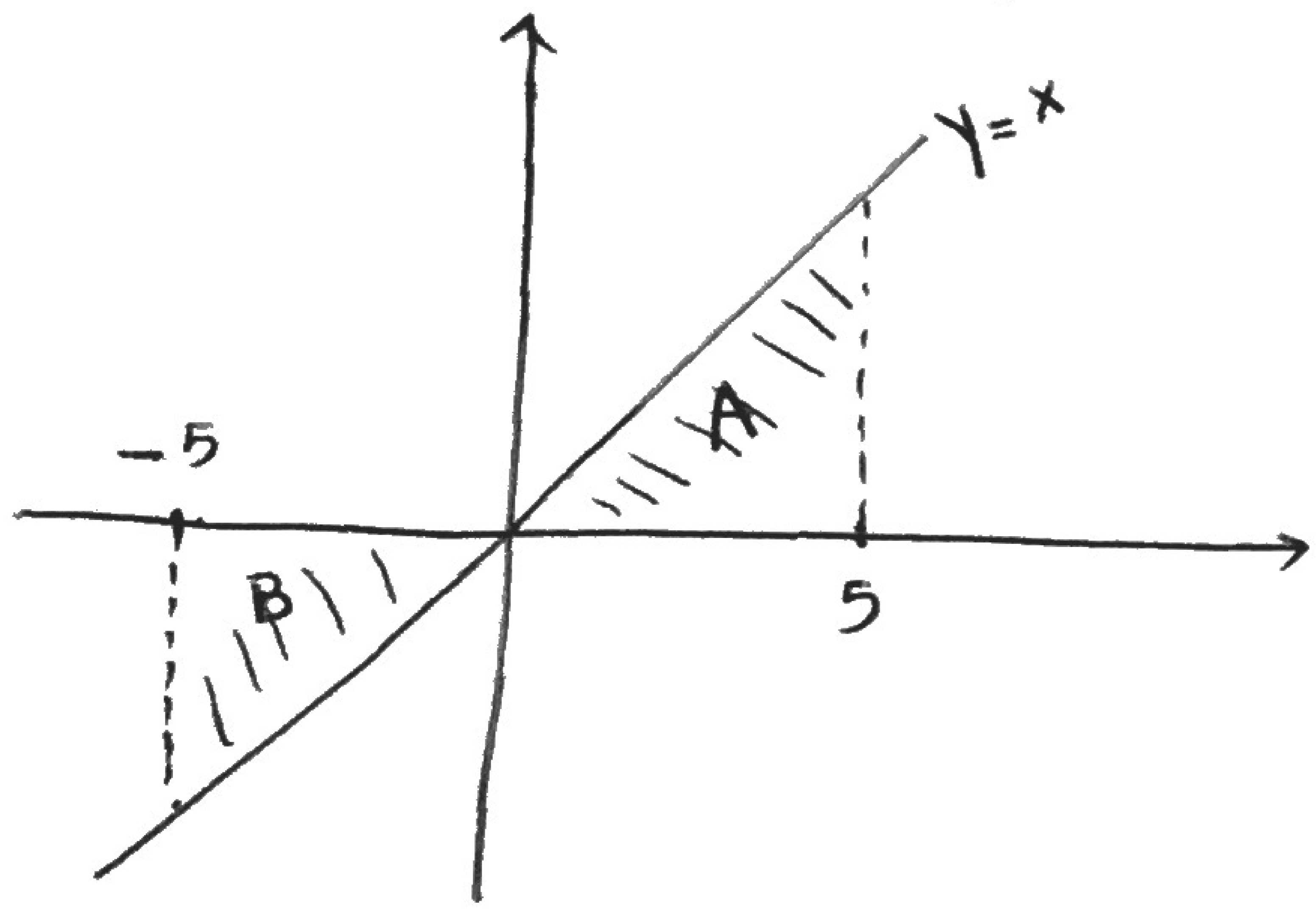
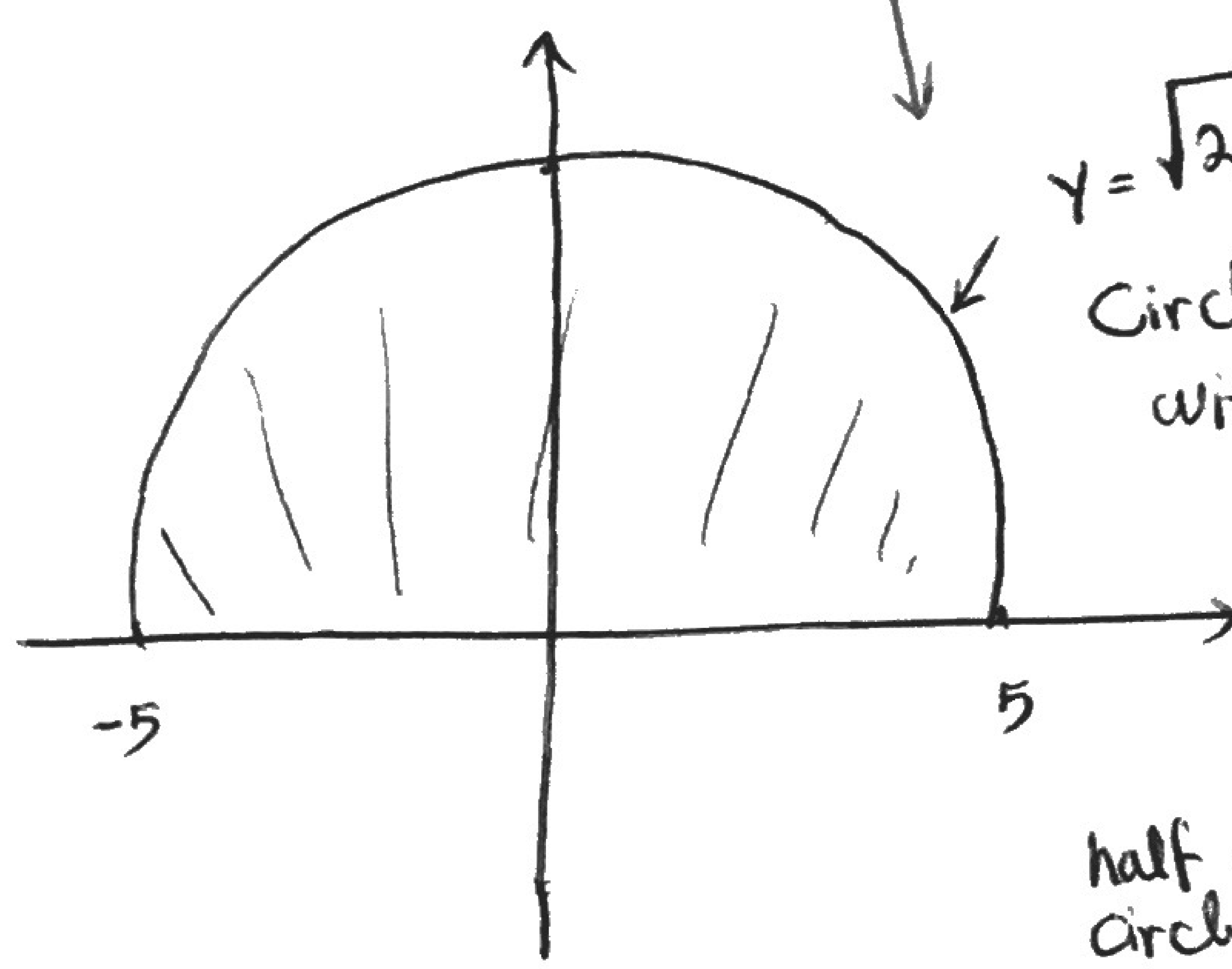
Homework 7

Section 5.2

38) $$\int_{-5}^{5}\left(x-\sqrt{25-x^2}\right)dx = \underbrace{\int_{-5}^{5} x\,dx}_{} - \underbrace{\int_{-5}^{5}\sqrt{25-x^2}\,dx}_{} \boxed{= -\frac{25}{2}\pi}$$





$y=\sqrt{25-x^2}$
Circle of radius 5 with center in origin

$$\int_{-5}^{5} x\,dx = \text{area}(A) - \text{area}(B) = 0$$

half of the area of a circle with radius 5

$$\int_{-5}^{5}\sqrt{25-x^2}\,dx = \frac{\pi\cdot 5^2}{2} = \frac{25\pi}{2}$$

71) First we divide $[0,1]$ into n subintervals of equal width $\Delta x = \frac{1}{n}$.

in any interval $\left[\frac{i-1}{n}, \frac{i}{n}\right]$ pick a rational point x_i^*. Then

$$\sum_{i=1}^{n} f(x_i^*)\,\Delta x = \sum_{i=1}^{n} 0\cdot\frac{1}{n} = 0$$

because all x_i^*'s are rational

Therefore $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\,\Delta x = 0$ when we pick x_i^*'s to be rational.

If in all subintervals $[\frac{i-1}{n}, \frac{i}{n}]$, we pick an irrational point x_i^*,

then

$$\sum_{i=1}^{n} f(x_i^*)\,\Delta x = \sum_{i=1}^{n} 1 \cdot \frac{1}{n} = 1$$

Therefore, when all x_i^* s are irrational $\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = 1$.

The limit depends on the possible choices of sample points. Therefore, f is not integrable.

(72) Divide $[0,1]$ into n subinterval with equal width. Pick $x_1^* = \frac{1}{n^2}$ and x_i^*

an arbitrary pt inside $[\frac{i-1}{n}, \frac{i}{n}]$ for any $2 \leq i \leq n$. Then

$$\sum_{i=1}^{n} f(x_i^*)\,\Delta x = \sum_{i=1}^{n} \frac{1}{x_i^*} \cdot \frac{1}{n} = \underbrace{\frac{1}{\frac{1}{n^2}}}_{\substack{\text{because} \\ x_1^* = \frac{1}{n^2}}} \cdot \frac{1}{n} + \sum_{i=2}^{n} \frac{1}{x_i^*} \cdot \frac{1}{n}$$

$$= n + \sum_{i=2}^{n} \frac{1}{x_i^*} \cdot \frac{1}{n} \geq n$$

$$\rightsquigarrow \sum_{i=1}^{n} f(x_i^*)\,\Delta x \geq n.$$

Section 5.3

60) $$g(x) = \int_{1-2x}^{1+2x} t\sin t \, dt = \int_{1-2x}^{1} t\sin t \, dt + \int_{1}^{1+2x} t\sin t \, dt$$

$$= \int_{1}^{1+2x} t\sin t \, dt - \int_{1}^{1-2x} t\sin t \, dt$$

Let $h(x) = \int_1^x t\sin t \, dt$. Therefore $h'(x) = x\sin x$, Now,

$$\int_1^{1+2x} t\sin t \, dt = h(1+2x) \rightsquigarrow \frac{d}{dx}\left(\int_1^{1+2x} t\sin t \, dt\right) = \frac{d}{dx}\left(h(1+2x)\right)$$

$$= h'(1+2x)\cdot \overset{2}{\cancel{\frac{d}{dx}(1+2x)}} = 2(1+2x)\sin(1+2x)$$

Similarly; $$\int_1^{1-2x} t\sin t \, dt = -2(1-2x)\sin(1-2x)$$

$$\Longrightarrow \boxed{g'(x) = 2(1+2x)\sin(1+2x) + 2(1-2x)\sin(1-2x)}$$

67) $x = 2 \rightsquigarrow F(2) = \int_2^2 e^{t^2} dt = 0 \rightsquigarrow$ line passes through the pt $(2,0)$.

Line is tangent to the curve $y = F(x)$. Therefore, its slope is equal to $F'(2)$.

$F'(x) = e^{x^2} \rightsquigarrow F'(2) = e^4 \Longrightarrow$ equ. of the line : $y = e^4(x-2)$ $\boxed{= e^4 x - 2e^4}$

Fundamental thm of Calc.

(83)

$$6 + \int_a^x \frac{f(t)}{t^2}\, dt = 2\sqrt{x} \quad \text{for all } x>0 \quad (*)$$

take the derivative of both sides

$$\frac{f(x)}{x^2} = \frac{1}{\sqrt{x}} \rightsquigarrow f(x) = \frac{x^2}{\sqrt{x}} = \boxed{x\sqrt{x}}$$

substitute $x = a$ in $(*)$ $\rightsquigarrow 6 = 2\sqrt{a} \rightsquigarrow \sqrt{a} = 3 \rightsquigarrow \boxed{a=9}$

Section 5.4

(49)

$$\int_0^2 (2y-y^2)\, dy = y^2 - \frac{1}{3}\cdot y^3 \Big|_0^2 = 2^2 - \frac{1}{3}\cdot 2^{3} = 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$$

Section 5.5

(31)

$$\int \frac{(\arctan x)^2}{x^2+1}\, dx = \int u^2\, du = \frac{u^3}{3} + C = \boxed{\frac{(\arctan x)^3}{3} + C}$$

$u = \arctan x \rightsquigarrow du = \frac{1}{x^2+1}\, dx$

(43)

$$\int \frac{dx}{\sqrt{1-x^2}\,\sin^{-1} x} = \int \frac{du}{u} = \ln|u| + C = \boxed{\ln|\sin^{-1} x| + C}$$

$u = \sin^{-1} x \rightsquigarrow du = \frac{1}{\sqrt{1-x^2}}\, dx$

(68)

$u(4)=9$

$$\int_0^4 \frac{x}{\sqrt{1+2x}}\, dx = \int_1^9 \frac{u-1}{4\sqrt{u}}\, du = \int_1^9 \left(\frac{\sqrt{u}}{4} - \frac{1}{4\sqrt{u}}\right) du = \frac{1}{6}u\sqrt{u} - \frac{1}{2}\sqrt{u}\Big|_1^9$$

$u(0)=1$

$u = 1+2x \rightsquigarrow du = 2dx$

$$= \left(\frac{27}{6} - \frac{3}{2}\right) - \left(\frac{1}{6} - \frac{1}{2}\right) = \boxed{\frac{10}{3}}$$

$3 \qquad -\frac{1}{3}$

(74) $f(-x) = \sin(\sqrt[3]{-x}) = \sin(-\sqrt[3]{x}) = -\sin\sqrt[3]{x} = -f(x) \Rightarrow \boxed{f \text{ is odd}}$

$$\int_{-2}^{3} \sin\sqrt[3]{x}\, dx = \underbrace{\int_{-2}^{2} \sin\sqrt[3]{x}\, dx}_{=\,0} + \int_{2}^{3} \sin\sqrt[3]{x}\, dx$$

For any $2 \leqslant x \leqslant 3$ we have $0 \leqslant \sqrt[3]{2} \leqslant \sqrt[3]{x} \leqslant \sqrt[3]{3} \leqslant \pi \approx 3.14$

$$\Downarrow$$

$$0 \leqslant \sin\sqrt[3]{x} \leqslant 1$$

$$\Downarrow$$

$$\underbrace{0\cdot(3-2)}_{=\,0} \leqslant \int_{2}^{3} \sin\sqrt[3]{x} \leqslant \underbrace{1\cdot(3-2)}_{=\,1}$$

■

(91) $$\int_{0}^{1} x^{a}(1-x)^{b}\, dx = -\int_{1}^{0} (1-u)^{a} u^{b}\, du = \int_{0}^{1} (1-u)^{a} u^{b}\, du = \int_{0}^{1} (1-x)^{a} x^{b}\, dx$$

$(u = 1-x \rightsquigarrow du = -dx)$

■

(92) $$\int_{0}^{\pi} x f(\sin x)\, dx = \int_{\pi}^{0} -(\pi-u)\, f(\sin(\pi-u))\, du = \int_{0}^{\pi} (\pi-u)\, f(\sin u)\, du$$

$u = \pi - x$

$\sin(\pi-u) = \sin u$

$$= \pi\int_{0}^{\pi} f(\sin u)\, du - \int_{0}^{\pi} u f(\sin u)\, du$$

These two integrals are equal

$$2\int_{0}^{\pi} x f(\sin x)\, dx = \pi\int_{0}^{\pi} f(\sin x)\, dx \rightsquigarrow \int_{0}^{\pi} x f(\sin x)\, dx = \frac{\pi}{2}\int_{0}^{\pi} f(\sin x)\, dx$$

■

(93)

$$\int_0^{\pi} \frac{x \sin x}{1+\underbrace{\cos^2 x}_{\substack{\shortparallel \\ 1-\sin^2 x}}}\,dx = \int_0^{\pi} x \cdot \frac{\sin x}{2-\sin^2 x}\,dx \underset{\substack{\uparrow \\ \text{ex 91}}}{=} \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{2-\sin^2 x}\,dx$$

$$\underset{\uparrow}{=} \frac{\pi}{2}\int_1^{-1} -\frac{1}{1+u^2}\,du = \frac{\pi}{2}\int_{-1}^{1} \frac{1}{1+u^2}\,du = \frac{\pi}{2}\Big(\underbrace{\tan^{-1}(1)-\tan^{-1}(-1)}_{\frac{\pi}{4}-(-\frac{\pi}{4})=\frac{\pi}{2}}\Big)$$

Substitute $u=\cos x \rightsquigarrow du=-\sin x\,dx$

$$\boxed{=\frac{\pi^2}{4}}$$

(94) (a)

$$\int_0^{\pi/2} f(\cos x)\,dx \underset{\uparrow}{=} \int_{\pi/2}^{0} -f\left(\cos\left(\frac{\pi}{2}-u\right)\right)du = \int_0^{\pi/2} f\left(\cos\left(\frac{\pi}{2}-u\right)\right)du$$

substitute $u=\frac{\pi}{2}-x$

$\rightsquigarrow du=-dx$

note that $\cos\left(\frac{\pi}{2}-u\right)=\sin u$

$$\rightsquigarrow \int_0^{\pi/2} f\left(\cos\left(\pi/2-u\right)\right)du = \int_0^{\pi/2} f(\sin u)\,du = \int_0^{\pi/2} f(\sin x)\,dx$$

(b)

$$\int_0^{\pi/2} \cos^2 x\,dx \underset{\uparrow}{=} \int_0^{\pi/2} \sin^2 x\,dx$$

because of part (a)

Moreover, $\sin^2 x+\cos^2 x=1 \rightsquigarrow \int_0^{\pi/2}(\sin^2 x+\cos^2 x)\,dx = \int_0^{\pi/2} 1\,dx = \pi/2$

$$\rightsquigarrow \int_0^{\pi/2} \sin^2 x\,dx + \int_0^{\pi/2} \cos^2 x\,dx = \pi/2 \rightsquigarrow \int_0^{\pi/2} \sin^2 x\,dx = \boxed{\int_0^{\pi/2} \cos^2 x\,dx = \pi/4}$$

Problem Plus (Chapter 5)

(3) $$\int_0^4 x e^{(x-2)^4} dx = \int_0^4 (x-2) e^{(x-2)^4} dx + \int_0^4 2e^{(x-2)^4} dx$$

$$= \int_0^4 (x-2) e^{(x-2)^4} dx + 2k$$

$$\int_0^4 (x-2) e^{(x-2)^4} dx \underset{\substack{u = x-2 \\ du = dx}}{=} \int_{-2}^{2} u e^{u^4} du = 0$$

because ue^{u^4} is an odd function.

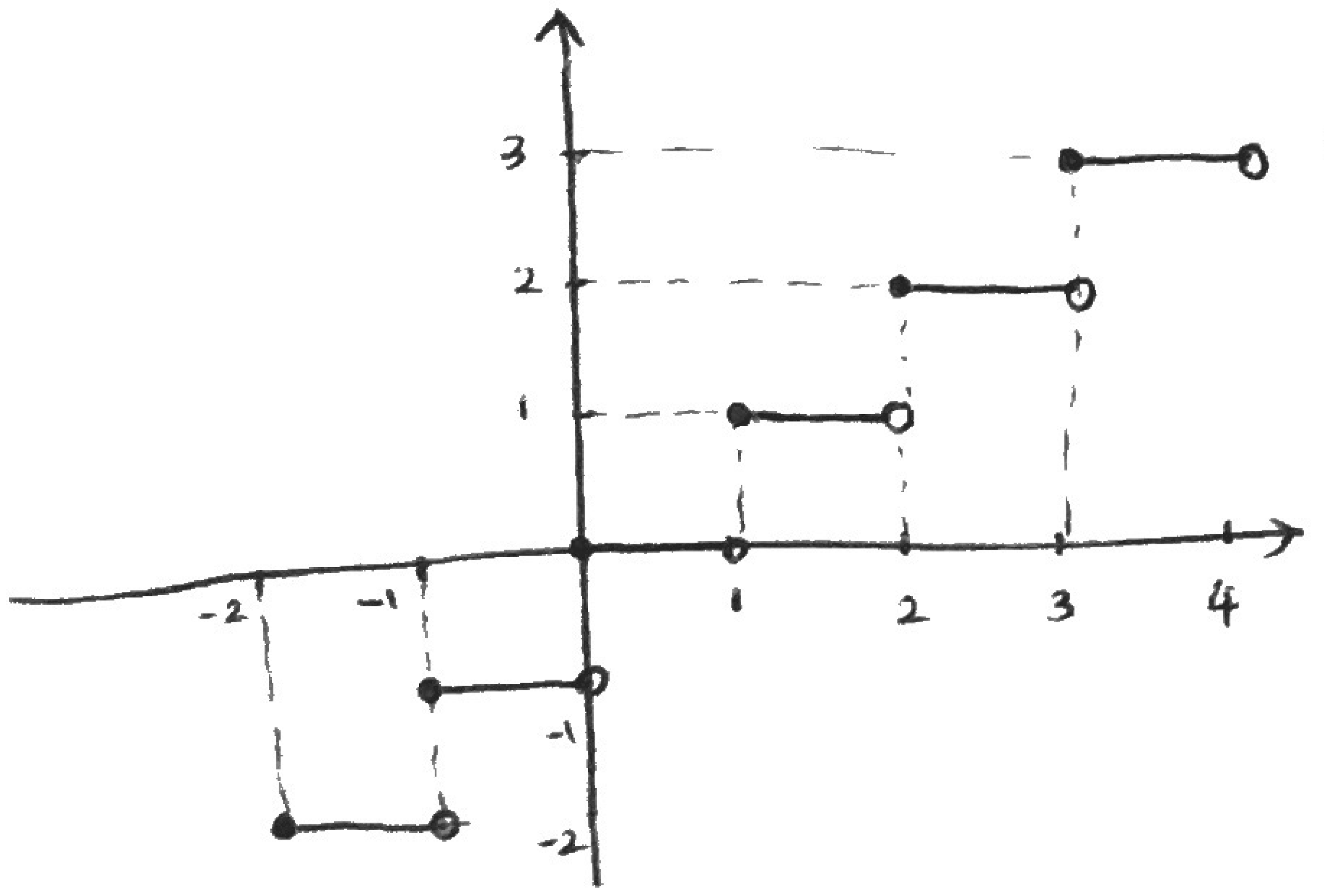
$$\Rightarrow \int_0^4 x e^{(x-2)^4} dx = 0 + 2k = \boxed{2k}$$

(9) $2 + x - x^2 = 0 \rightsquigarrow x = -1,\ x = 2$ moreover, $2+x-x^2 \geq 0$ for any $-1 \leq x \leq 2$

$2+x-x^2 < 0$ for any $x < -1$ and $x > 2$

Therefore, $\int_a^b 2+x-x^2$ is maximum if $a=-1$, $b=2$.

(10) $f(x) = [x]$ is the largest integer not greatest than x. Its graph is as ~~the~~ follows.



(a) $$\int_0^a [x]dx = \int_0^1 \overset{0}{\cancel{[x]}}dx + \int_1^2 \overset{1}{\cancel{[x]}}dx + \int_2^3 \overset{2}{\cancel{[x]}}dx + \cdots$$

$$+ \int_{a-1}^a \overset{a-1}{\cancel{[x]}}dx = 1 + 2 + 3 + \cdots + a-1$$

$$= \frac{(a-1)a}{2}$$

(b) $$\int_a^b [x]\,dx = \int_a^{[a]+1} \underset{[a]}{[x]}\,dx + \int_{[a]+1}^{[a]+2} \underset{[a]+1}{[x]}\,dx + \cdots + \int_{[b]-1}^{[b]} \underset{[b]-1}{[x]}\,dx + \int_{[b]}^{b} \underset{[b]}{[x]}\,dx$$

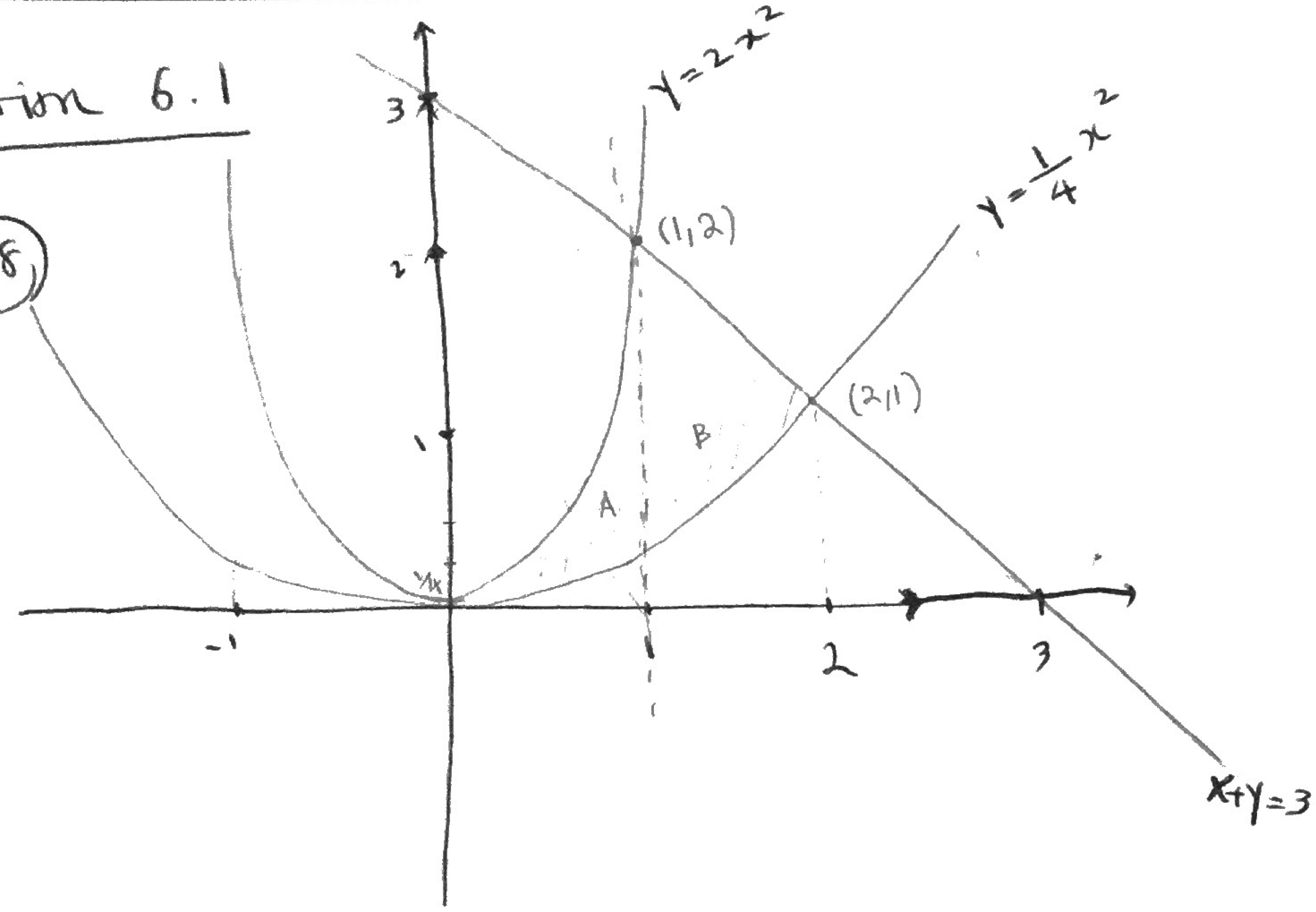
$$= [a]\big([a]+1-a\big) + \underbrace{\big([a]+1\big) + \big([a]+2\big) + \cdots + \big([b]-1\big)}_{\frac{([a]+[b])([b]-[a]-1)}{2}} + [b]\big(b-[b]\big)$$

$$= [a]^2 + [a] - a[a] + \underbrace{\frac{([a]+[b])([b]-[a]-1)}{2}}_{\frac{1}{2}([a][b]+[b]^2-[a]^2-[a][b]-[a]-[b])} + b[b] - [b]^2$$

$$= \frac{1}{2}[a]^2 + \frac{1}{2}[a] - a[a] - \frac{1}{2}[b]^2 - \frac{1}{2}[b] + b[b].$$

Section 6.1

(28)



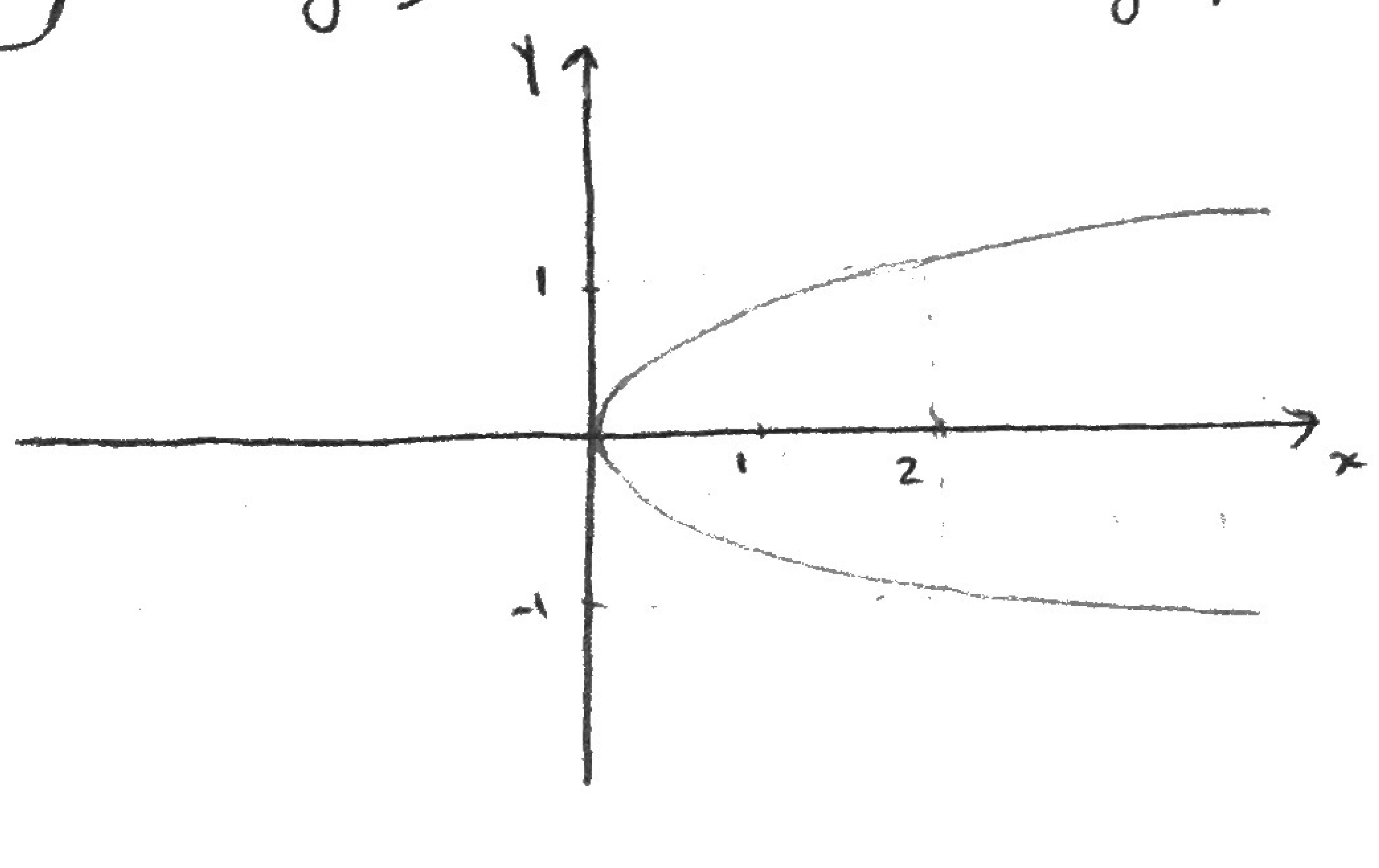
The area between the curves $y=2x^2$, $y=\frac{x^2}{4}$ and $x+y=3$ is equal to the sum of the area between $y=\frac{x^2}{4}$ and $y=2x^2$ from 0 to 1 (area of the domain A) and the area between $y=\frac{x^2}{4}$ and $x+y=3$ from 1 to 2 (area of the domain B).

Therefore,

$$\underbrace{\int_0^1 \left(2x^2-\frac{x^2}{4}\right)dx}_{\text{area }(A)} + \underbrace{\int_1^2 \left((3-x)-\frac{x^2}{4}\right)dx}_{\text{area }(B)} = \left(\frac{2}{3}x^3-\frac{x^3}{12}\right)\Big|_0^1 + \left(3x-\frac{x^2}{2}-\frac{x^3}{12}\right)\Big|_1^2$$

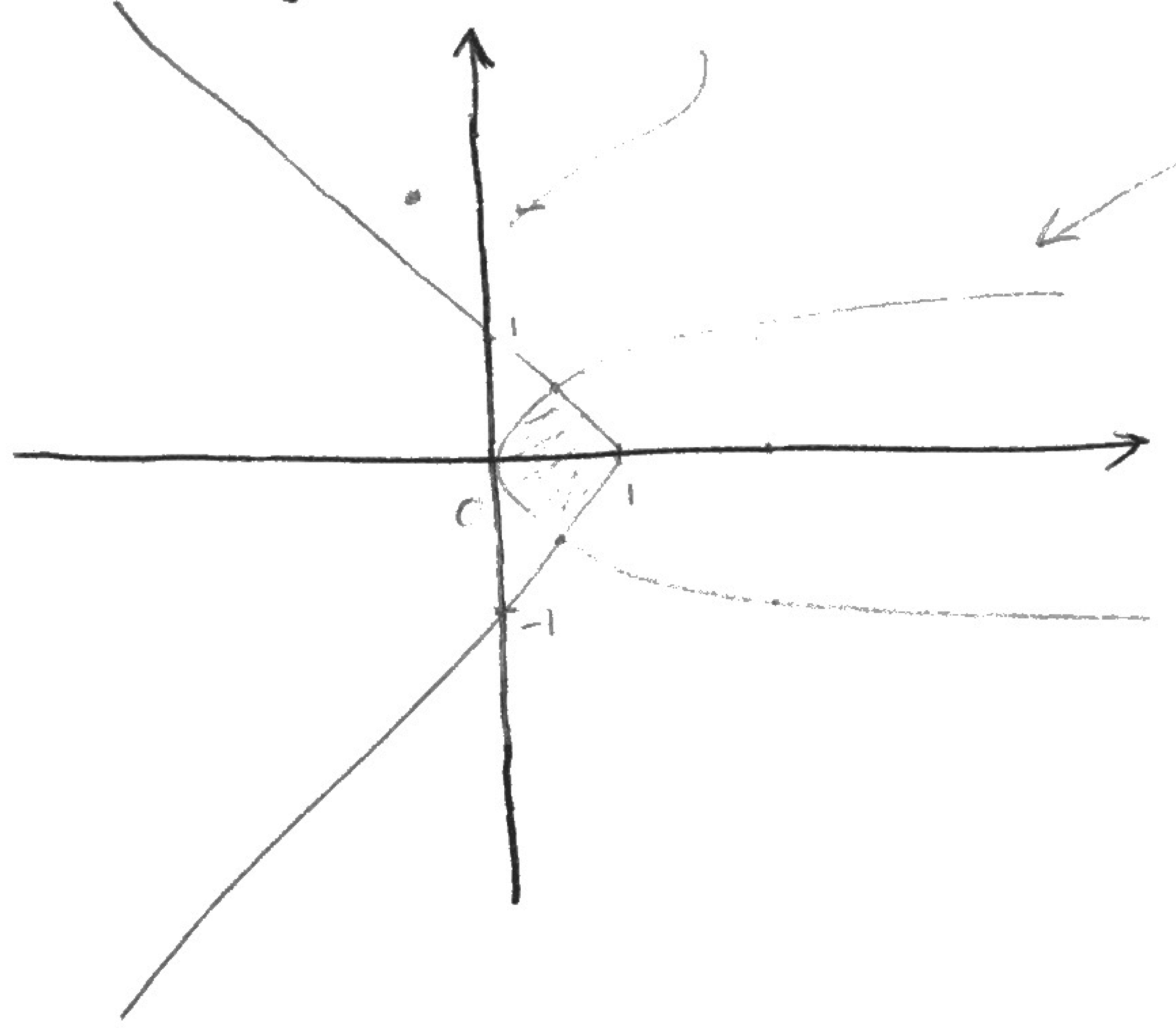
$$=\left(\frac{2}{3}-\frac{1}{12}\right)+\left(6-2-\frac{8}{12}-3+\frac{1}{2}+\frac{1}{12}\right) = \frac{18}{12} = \boxed{\frac{3}{2}}$$

(46) $x-2y^2\geq 0 \rightsquigarrow$ Sketch the graph of $x=2y^2$, since $x-2y^2$ is negative at point $(0,1)$, then the ~~ax~~ region of $x-2y^2\geq 0$ is ~~in~~ the right hand-side of the graph $x=2y^2$.



$1-x-|y|=0 \rightsquigarrow x = 1-|y|$

at $(0,0)$ we have $1-x-|y|\geq 0$ therefore, the region $1-x-|y|\geq 0$ is the left hand side of the graph $1-x-|y|=0$



Intersection pts of the graphs:

$x = 2y^2 = 1-|y|$

$\rightsquigarrow 2y^2+|y|-1 = 0$

$\rightsquigarrow (2|y|-1)(|y|+1) = 0$

$\rightsquigarrow |y| = \frac{1}{2} \rightsquigarrow y = +\frac{1}{2}$ & $-\frac{1}{2}$

$\rightsquigarrow (\frac{1}{2}, \frac{1}{2})$ $(\frac{1}{2}, -\frac{1}{2})$

$$\text{Area} = \int_0^{1/2} [\underbrace{\sqrt{\tfrac{x}{2}} - (-\sqrt{\tfrac{x}{2}})}_{2\sqrt{\frac{x}{2}}}]\,dx + \int_{1/2}^{1} [\underbrace{1-x-(x-1)}_{2-2x}]\,dx$$

$$= \frac{2\sqrt{2}}{3} x^{3/2} \Big|_0^{1/2} + (2x-x^2)\Big|_{1/2}^{1} = \frac{1}{3} + (1-\frac{3}{4}) = \boxed{\frac{7}{12}}$$

6.2

(42)

$$\pi \int_1^4 [3^2 - (3-\sqrt{x})^2]\,dx$$

You can describe ~~different solids~~ ~~(more than one)~~ the solid, with that the above integral represents its volume, in different ways,

For example, the solid Constructed by revolving the region between

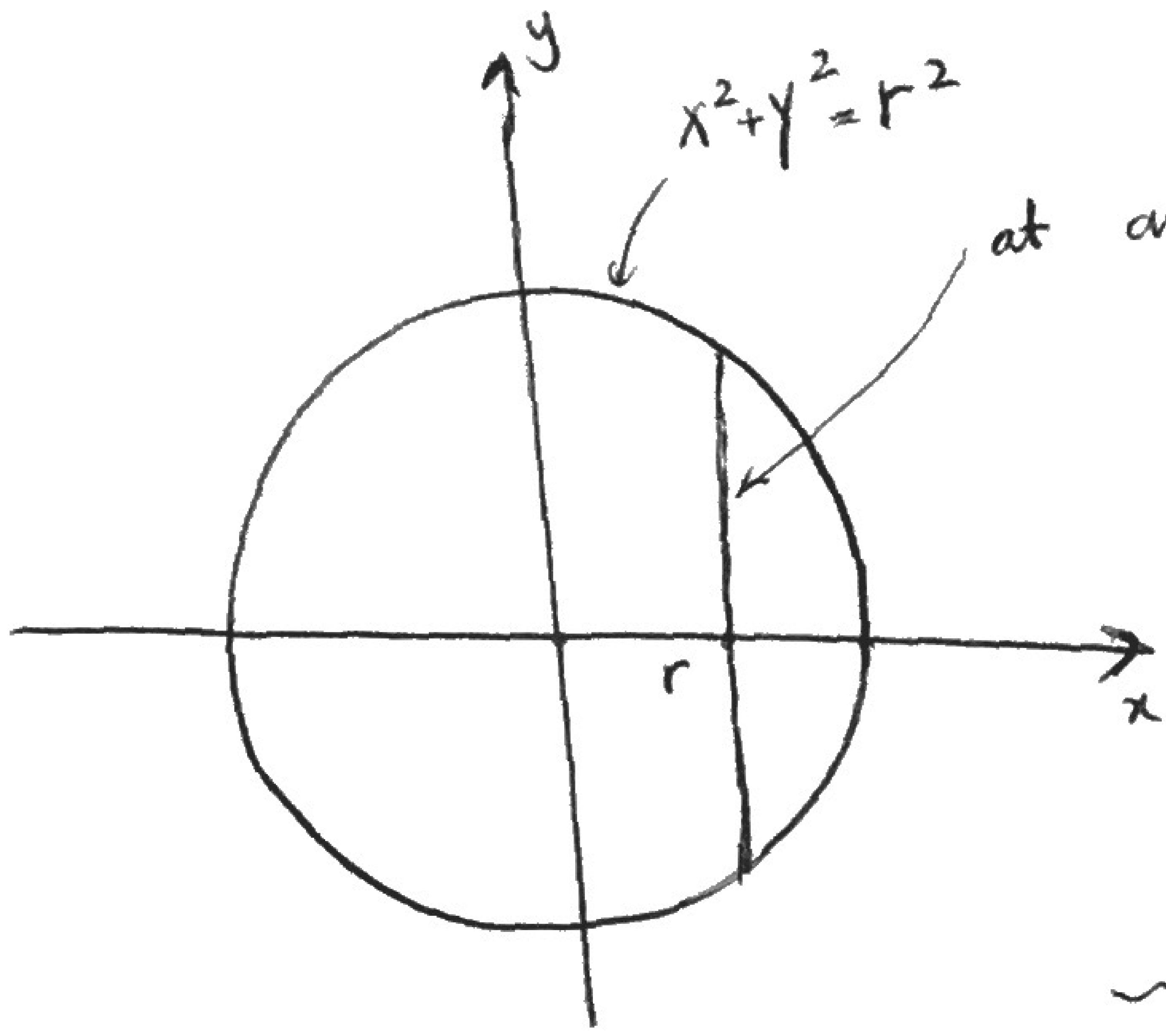
$y=0$, $y=-\sqrt{x}$, $x=1$, $x=4$

about the $y=-3$ line.

or, the solid Constructed by revolving the region between

$y=3$, $y=3-\sqrt{x}$ $x=1$, $x=4$ about $y=0$

54



at any $-r \leq x \leq r$, the cross section is a square.

$\rightsquigarrow$ edge of the ~~eq~~ square $= 2\cdot\sqrt{r^2-x^2}$

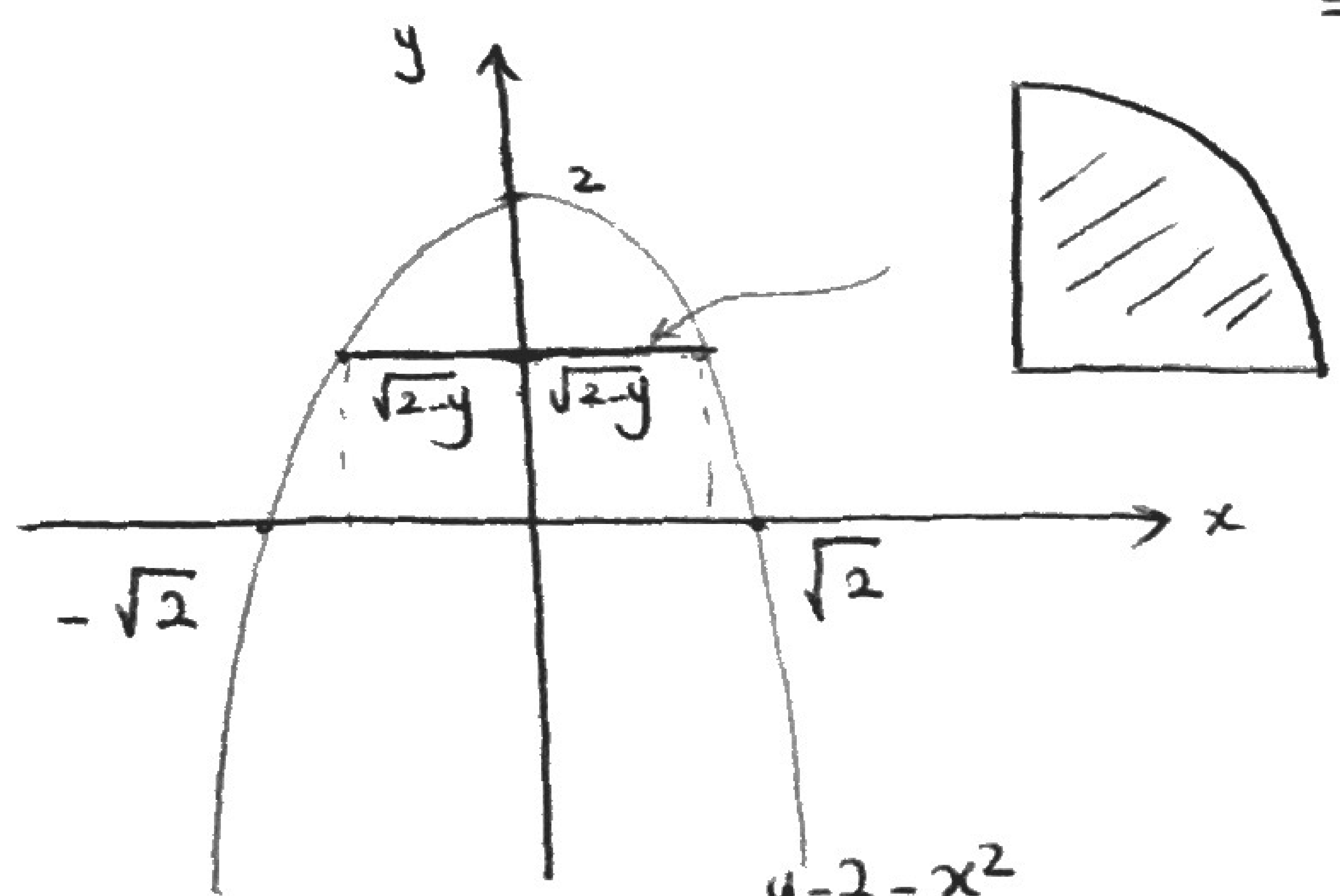
$\rightsquigarrow$ area $= \left(2\cdot\sqrt{r^2-x^2}\right)^2$

$= 4\cdot(r^2-x^2)$

$$\rightsquigarrow \int_{-r}^{r} A(x)dx = \int_{-r}^{r} 4(r^2-x^2)dx = 4r^2x - \frac{4x^3}{3}\Big|_{-r}^{r}$$

$$= \left(4r^3-\frac{4}{3}r^3\right)-\left(-4r^3+\frac{4}{3}r^3\right) = \boxed{\frac{16}{3}r^3}$$

60



radius of the quarter-circle at y is equal to : $2\cdot\sqrt{2-y}$

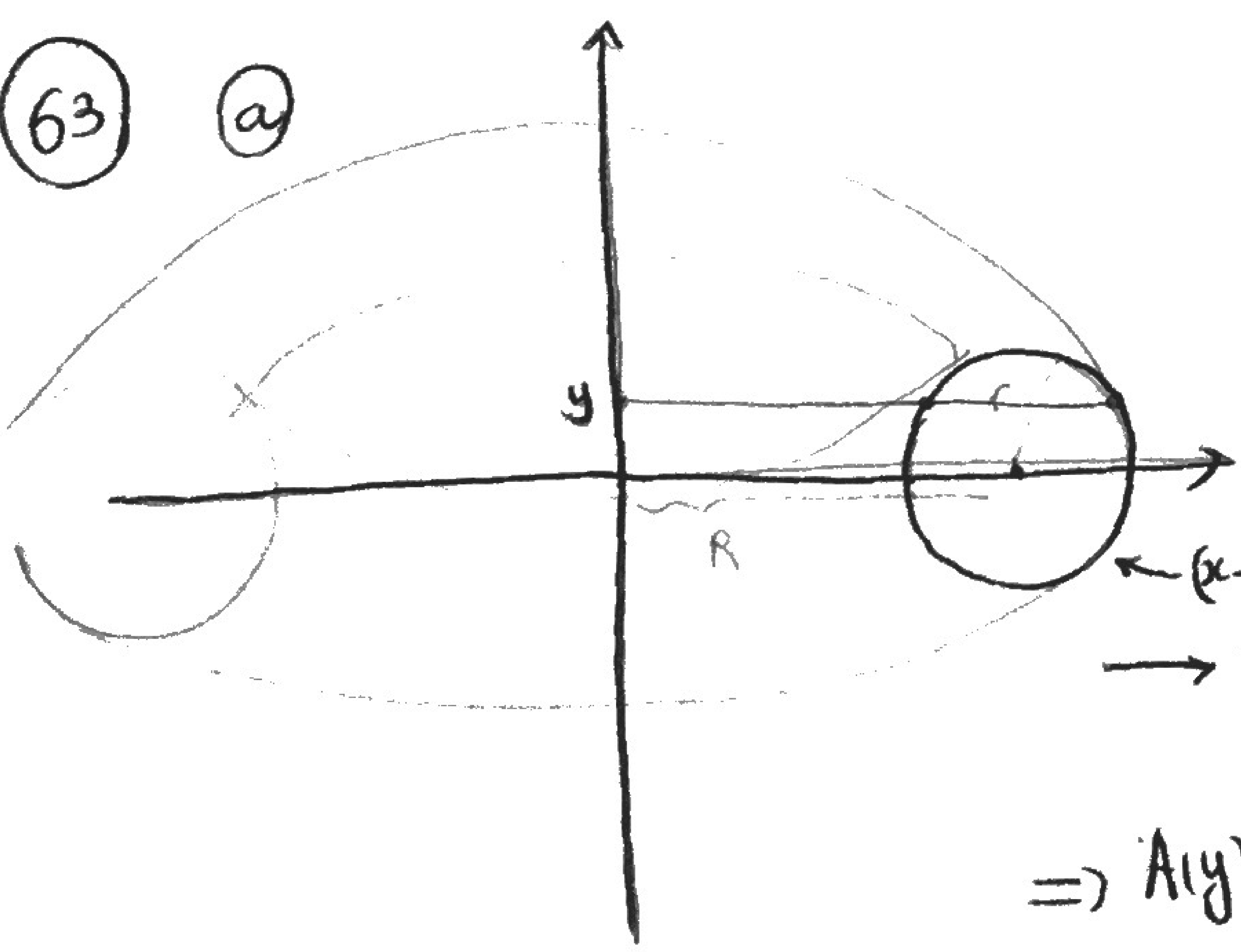
$\rightsquigarrow A(y) = \frac{1}{4}\pi\left(2\cdot\sqrt{2-y}\right)^2$

$= \frac{\pi}{4}\cdot 4(2-y) = \pi(2-y)$

$$\rightsquigarrow \int_0^2 \pi(2-y)dy = 2\pi y - \frac{\pi}{2}y^2\Big|_0^2$$

$$= 4\pi - 2\pi = \boxed{2\pi}$$

63 a



The torus is obtained by revolving a disk of radius "r" with center at $(R,0)$ around y-axis. at each point y, the slice has the shape of a washer as below.

$\leftarrow (x-R)^2+y^2=r^2$

$\longrightarrow x = R \pm \sqrt{r^2-y^2}$

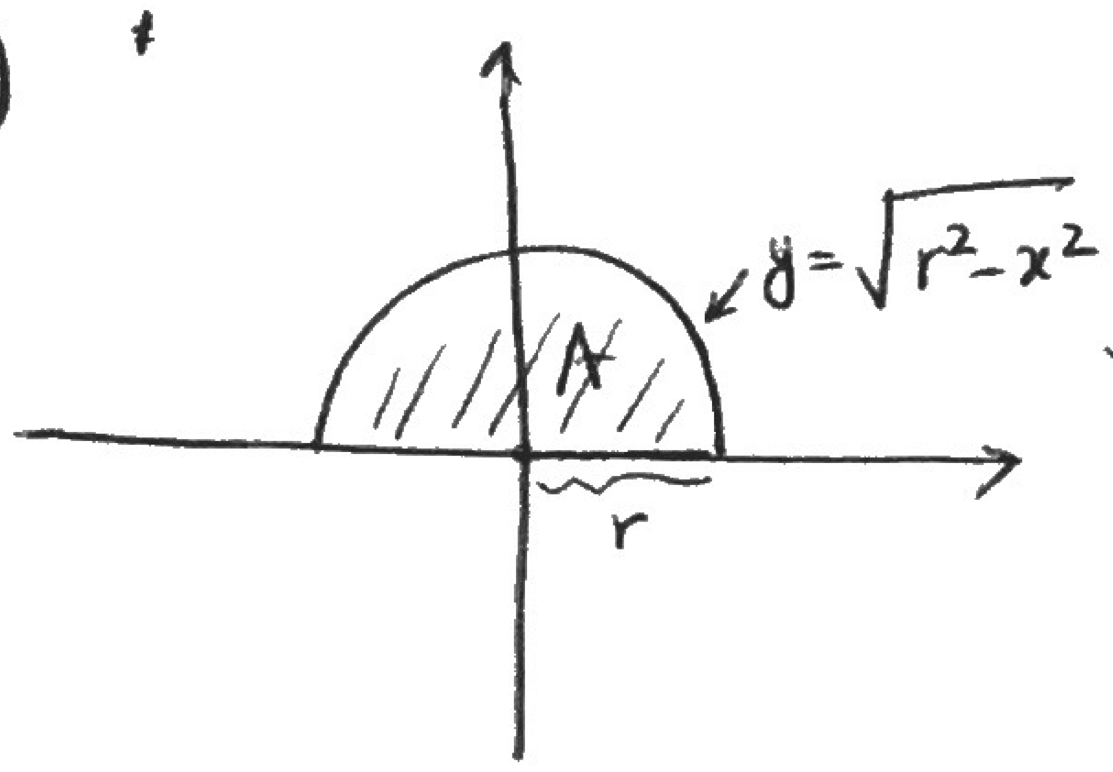
$\rightsquigarrow R_{out} = R+\sqrt{r^2-y^2}$

$R_{in} = R-\sqrt{r^2-y^2}$

$$\Rightarrow A(y) = \pi\left(R+\sqrt{r^2-y^2}\right)^2 - \pi\left(R-\sqrt{r^2-y^2}\right)^2 = 4\pi R\sqrt{r^2-y^2}$$

$$\text{Volume} = \int_{-r}^{r} 4\pi R \sqrt{r^2 - y^2}\, dy$$

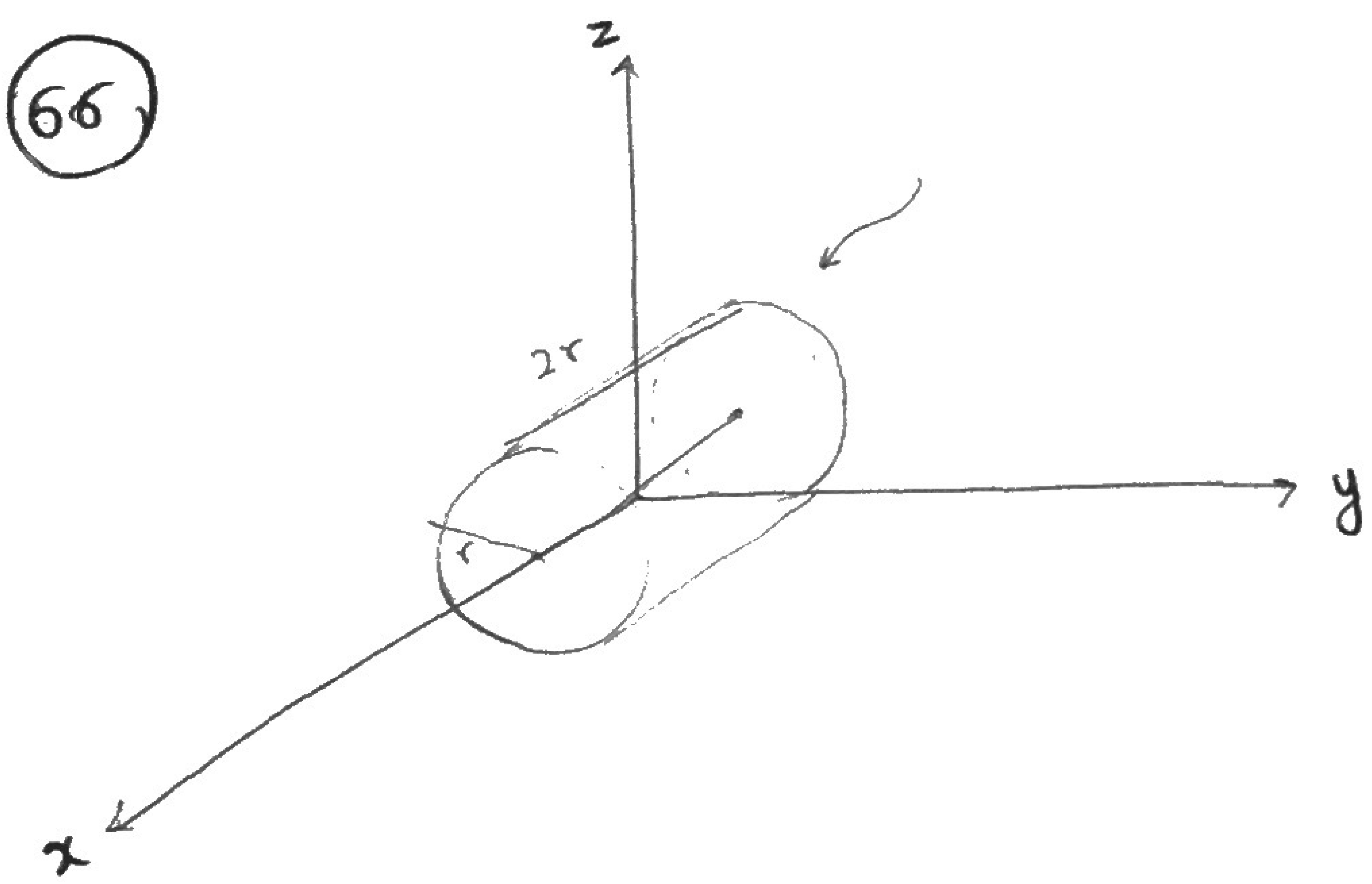
(b)



$$\rightsquigarrow \int_{-r}^{r} \sqrt{r^2-x^2}\, dx = \text{area}(A) = \frac{\pi r^2}{2}$$

$$\rightsquigarrow \int_{-r}^{r} 4\pi R\sqrt{r^2-y^2}\, dy = 4\pi R \underbrace{\int_{-r}^{r} \sqrt{r^2-y^2}\, dy}_{\frac{\pi r^2}{2}} = \boxed{2\pi^2 r^2 R}$$

(66)



put the axe of the first cylinder over the x-axis such that its mid pt. is at origin. The coordinates of the pts (x,y,z) inside this cylinder satisfies the following relations:

$$-r \leq x \leq r \quad \text{and} \quad \underbrace{y^2+z^2 \leq r^2}_{}$$

The pts inside a disk with radius r with center at origin in yz-plane

Similarly, we can put the axe of the second cylinder over the y-axis such that its mid pt is on origin. The coordinates of the pts inside this cylinder, satisfy the following relations: $-r \leq y \leq r$ and $x^2+z^2 \leq r^2$

Therefore, their intersection consists of the pts (x,y,z) whose coordinates satisfies the following condition:

$$-r \leq x \leq r \qquad y^2+z^2 \leq r^2$$
$$-r \leq y \leq r \qquad x^2+z^2 \leq r^2$$

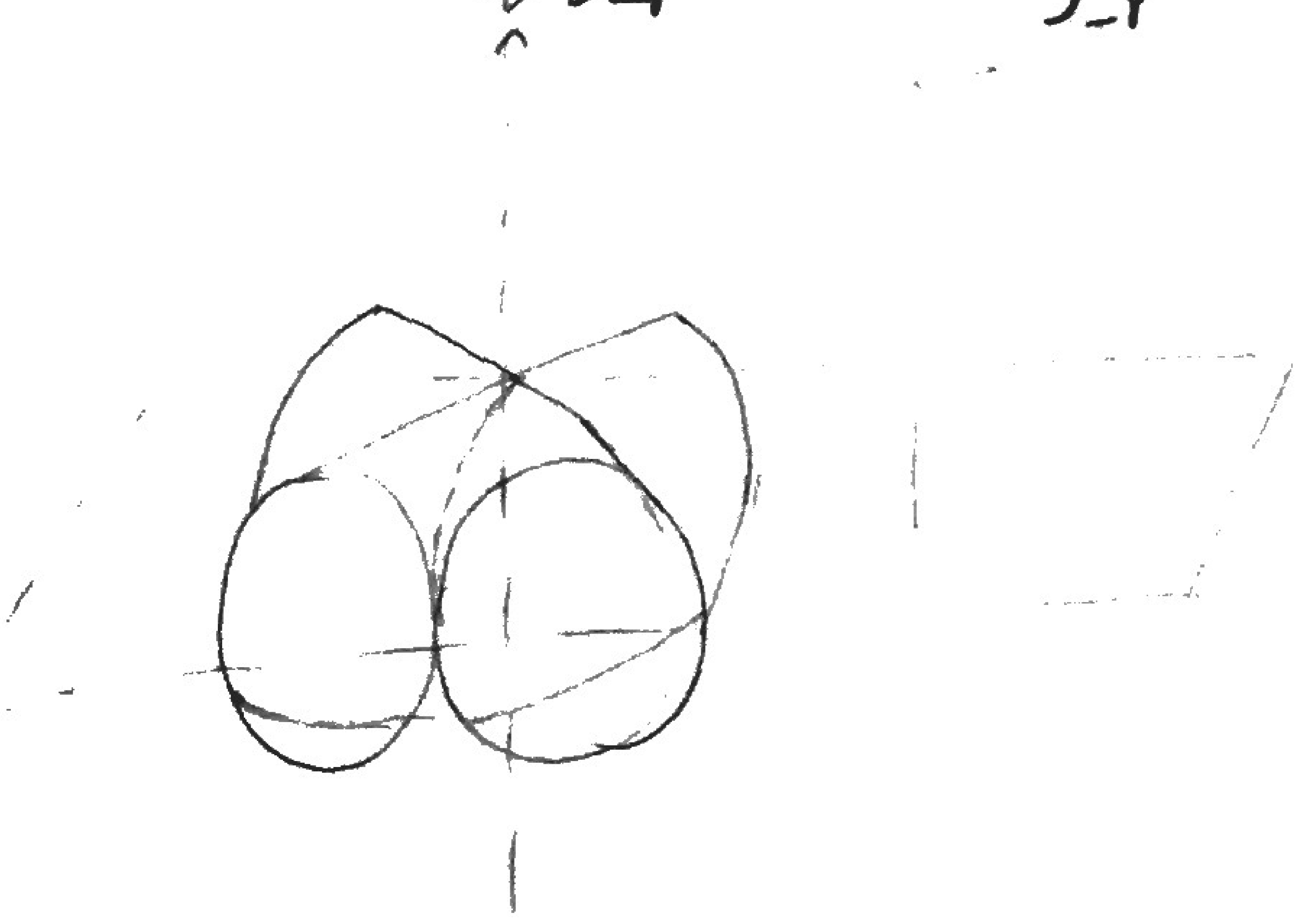
If we slice this solid, with planes perpendicular to z-axis the intersection consists of the pts (x,y,z) where z $\quad y^2, x^2 \leq r^2-z^2 \rightsquigarrow -\sqrt{r^2-z^2} \leq x, y \leq \sqrt{r^2-z^2}$

Therefore, it's a square, with edge length $2\sqrt{r^2-z^2}$.

$$\longrightarrow A(z) = \left(2\sqrt{r^2-z^2}\right)^2 = 4(r^2-z^2)$$

$$\longrightarrow \text{Vol} = \int_{-r}^{r} A(z)\,dz = \int_{-r}^{r} (4r^2-4z^2)\,dz = 4r^2z - \frac{4z^3}{3}\Big|_{-r}^{r} = \left(4r^3 - \frac{4}{3}r^3\right) - \left(-4r^3 + \frac{4}{3}r^3\right)$$

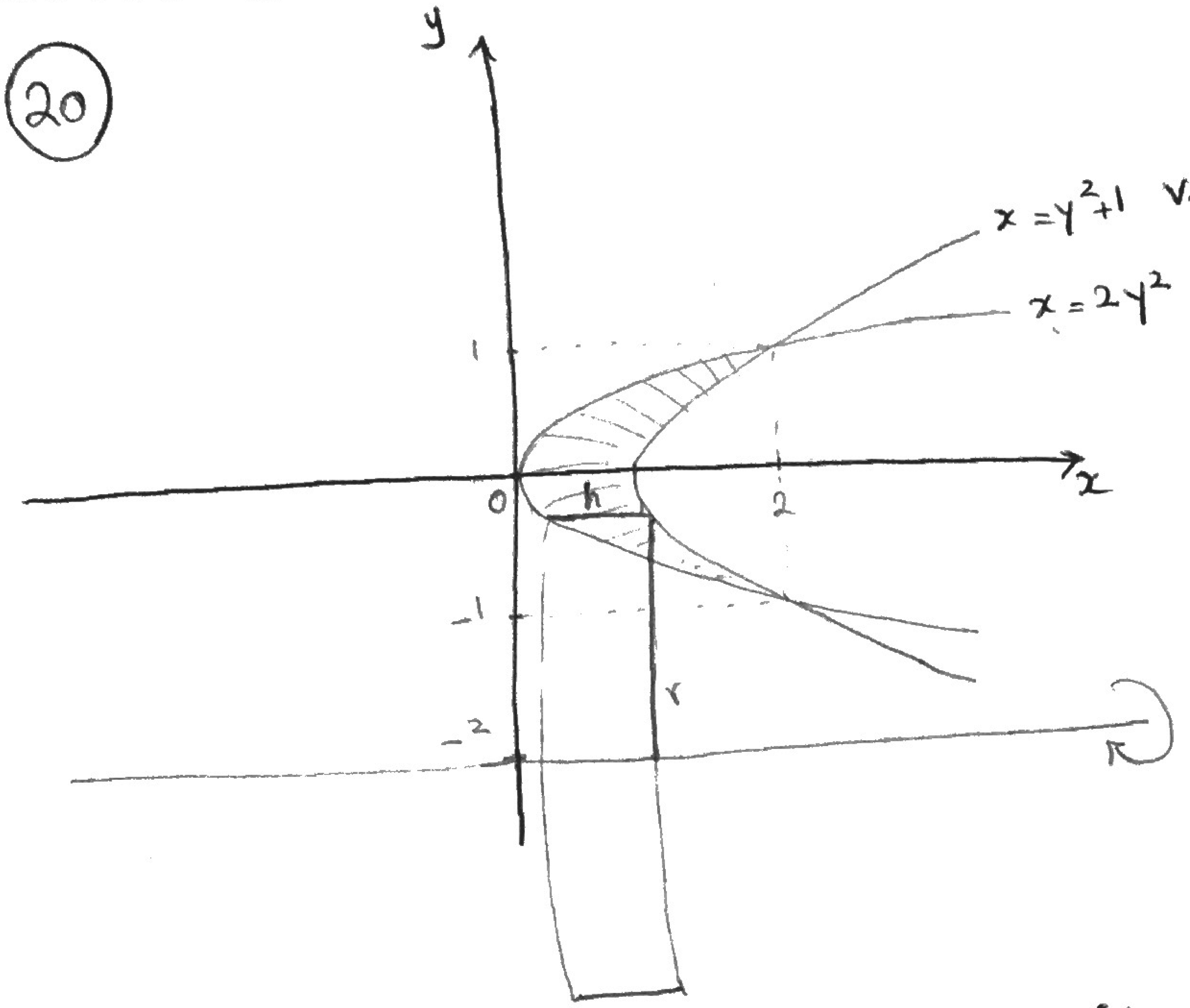
$$= \boxed{\frac{16}{3}r^3}$$



← planes perpendicular to z axis cut the intersection region in squares!

Section 6.3

(20)



$x = y^2+1$ vertex at $(1,0)$, passing $(2,1), (2,-1)$

$x = 2y^2$ ⇝ parabola with vertex at $(0,0)$ & passing $(2,1)$ & $(2,-1)$

$\leadsto h = y^2+1-2y^2 = -y^2+1$

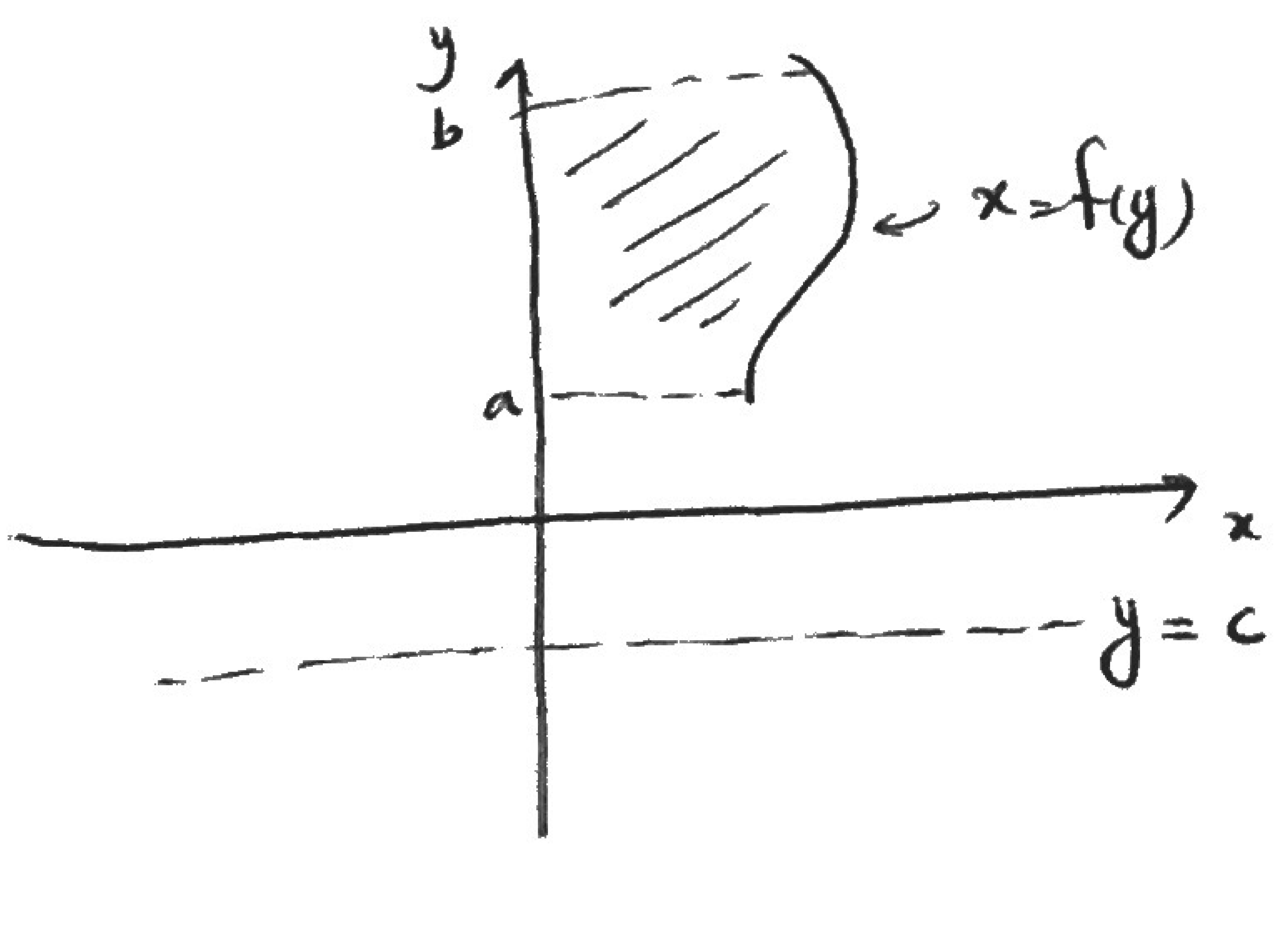
$r = \underbrace{y+2}_{y-(-2)}$

$$\leadsto \text{Vol} = \int_{-1}^{1} 2\pi \underbrace{(y+2)(-y^2+1)}_{-y^3-2y^2+y+2}\,dy = \int_{-1}^{1} 2\pi(-y^3-2y^2+y+2)\,dy$$

$$= 2\pi\left(-\frac{y^4}{4} - \frac{2y^3}{3} + \frac{1}{2}y^2 + 2y\right)\Bigg|_{-1}^{1} = 2\pi\left(-\tfrac{1}{4} - \tfrac{2}{3} + \tfrac{1}{2} + 2 - \left(-\tfrac{1}{4} + \tfrac{2}{3} + \tfrac{1}{2} - 2\right)\right)$$

$$= 2\pi\left(\frac{8}{3}\right) = \boxed{\frac{16}{3}\pi}$$

(31) $2\pi \displaystyle\int_1^4 \frac{y+2}{y^2}\,dy$,

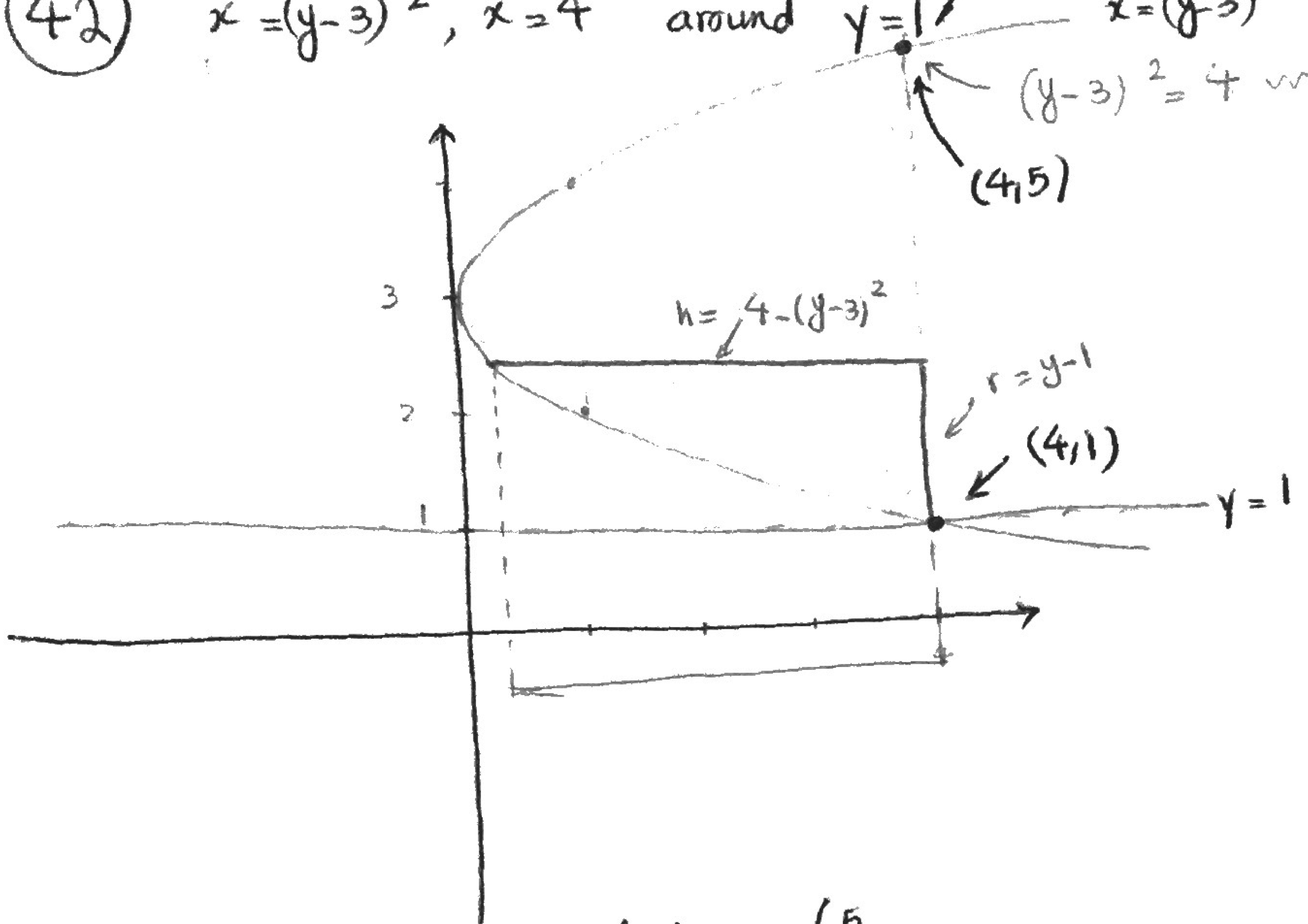


⇝ Compute the volume of the solid obtained by revolving the region between $x = f(y)$ and y-axis from a to b around $y = c$. Using cylindrical shells method we get:

$$\text{Vol} = \int_a^b 2\pi(y-c)\cdot f(y)\,dy$$

Therefore, $2\pi\displaystyle\int_1^4 \frac{y+2}{y^2}\,dy$ represents the volume of the solid obtained by revolving the region between $x = \frac{1}{y^2}$, y-axis, $y = 1$ and $y = 4$ about the line $y = -2$.

(42) $x=(y-3)^2$, $x=4$ around $y=1$



$(y-3)^2=4 \rightsquigarrow \begin{cases} y-3=2 \rightsquigarrow y=5 \\ 3-y=2 \rightsquigarrow y=1 \end{cases}$

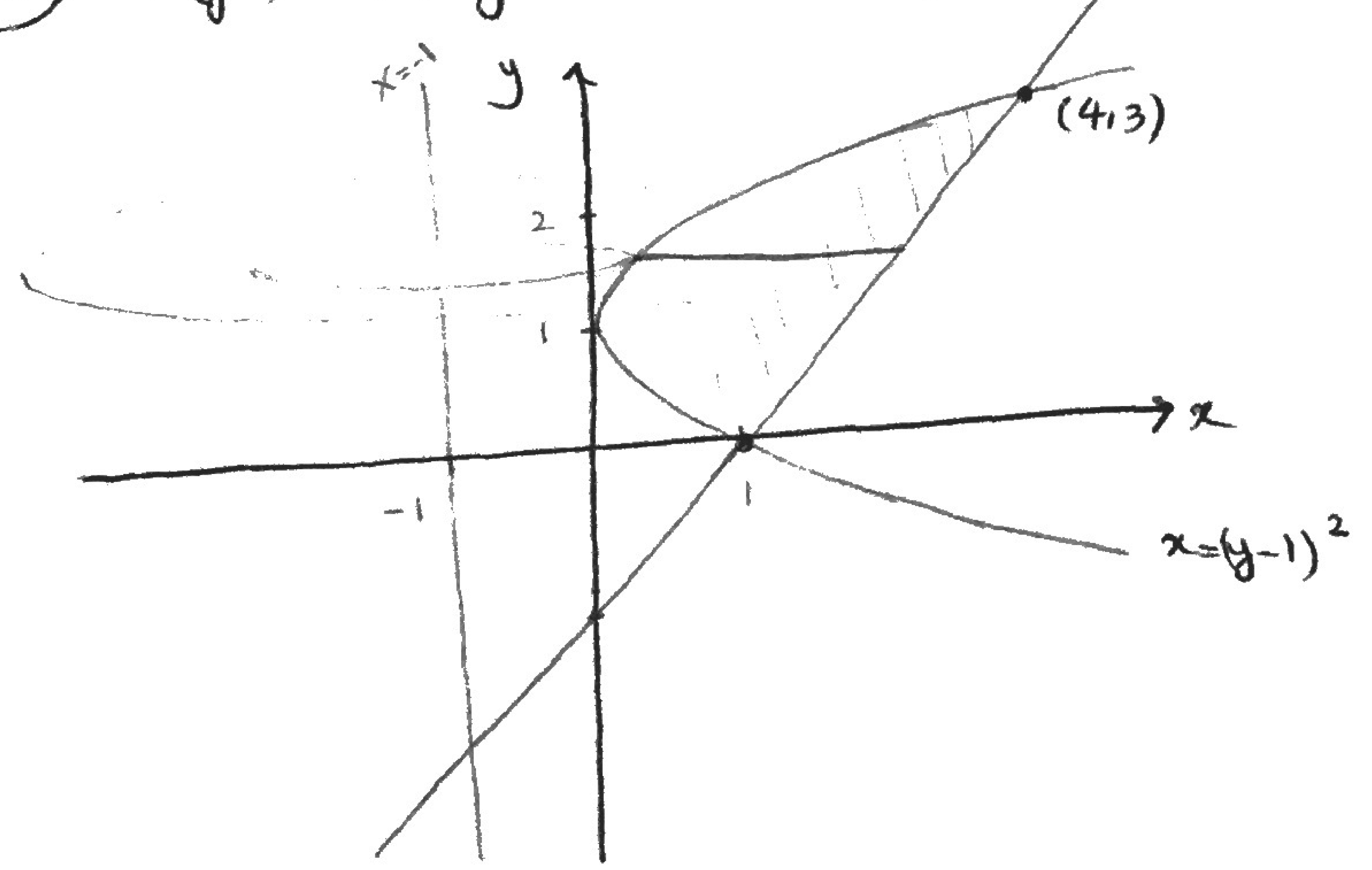
Using cylindrical shell method: $\displaystyle\int_1^5 2\pi(y-1)\big(\underbrace{4-(y-3)^2}_{-y^2+6y-5}\big)\,dy = \int_1^5 2\pi\left(-y^3+\underbrace{6y^2+y^2}_{7y^2}\underbrace{-5y-6y}_{-11y}+5\right)$

$$= 2\pi\left(-\frac{y^4}{4}+\frac{7}{3}y^3-\frac{11}{2}y^2+5y\right)\Big|_1^5 = 2\pi\left[-\frac{5^4}{4}+\frac{7}{3}5^3-\frac{11}{2}25+25-\underbrace{\left(-\frac{1}{4}+\frac{7}{3}-\frac{11}{2}+5\right)}_{\frac{-3+28-66+60}{12}}\right]$$

$$= 2\pi\left[-\frac{625}{4}+\frac{875}{3}-\frac{275}{2}+25-\frac{19}{12}\right] = 2\pi\cdot\frac{-\overset{1875}{3(625)}+\overset{3500}{4(875)}-\overset{1650}{6(275)}+300-19}{12}$$

$$= 2\pi\cdot\frac{256}{12} = \boxed{\frac{128\pi}{3}}$$

(43) $x=(y-1)^2$, $x-y=1$ about $x=-1$



Intersection pts:

$(y-1)^2=y+1 \rightsquigarrow y^2-2y+\cancel{1}=y+\cancel{1}$

$\rightsquigarrow y^2-3y=0$

$\rightsquigarrow y=0$ or $y=3$

$\rightsquigarrow (1,0)$ and $(4,3)$

Slicing method : For any $0 \leq y \leq 3$, the cross sectional slice is a washer with outer radius ~~[illegible]~~ $y+1-(-1) = y+2$ and inner radius $(y-1)^2-(-1) = (y-1)^2+1$

Therefore : $A(y) = \pi(y+2)^2 - \pi\left(\underbrace{(y-1)^2+1}_{y^2-2y+2}\right)^2$

$$= \pi\left[y^2+4y+4-(y^4-4y^3+8y^2-8y+4)\right]$$

$$= \pi\left[-y^4+4y^3-7y^2+12y\right]$$

$$\rightsquigarrow \text{Vol} = \int_0^3 \pi\left[-y^4+4y^3-7y^2+12y\right]dy = \pi\left[-\frac{y^5}{5}+y^4-\frac{7}{3}y^3+6y^2\right]_0^3$$

$$= \pi\left[-\frac{\overset{243}{3^5}}{5}+\overset{81}{3^4}-\overset{63}{\frac{7}{3}\cdot 3^3}+\overset{54}{6\cdot 3^2}\right] = \pi\cdot\boxed{\frac{117}{5}}$$