Section 9.1

4 (a) $y = \cos kt \longrightarrow y' = -k\sin kt \longrightarrow y'' = -k^2 \cos kt$

$\rightsquigarrow 4y'' = -4k^2 \cos kt = -25 \cos kt$

$\rightsquigarrow -4k^2 = -25$

$\rightsquigarrow 4k^2 = 25 \rightsquigarrow k = \pm \frac{5}{2}$

(b) $y = A\sin kt + B\cos kt \rightsquigarrow y' = Ak\cos kt - Bk\sin kt$

$\rightsquigarrow y'' = -Ak^2 \sin kt - Bk^2 \cos kt$

$\rightsquigarrow y'' = -A\cdot\frac{25}{4}\sin kt - B\,\frac{25}{4}\cos kt$

$= \frac{25}{4}(-A\sin kt - B\cos kt)$

$= -\frac{25}{4}y$

$\rightsquigarrow 4y'' = -25y$

12 A. is incorrect, because for $x>0, y>0$ $y'>1$, but the graph of the function has $-$ slope for point with $x, y>0$.

B. is incorrect, because for $x=0$, $y'=0$, but at the intersection of the graph with y axis, $y'>0$.

C. The graph, can be a solution of $y' = 1-2xy$.

when $x=0$, $y'=1$ can be true

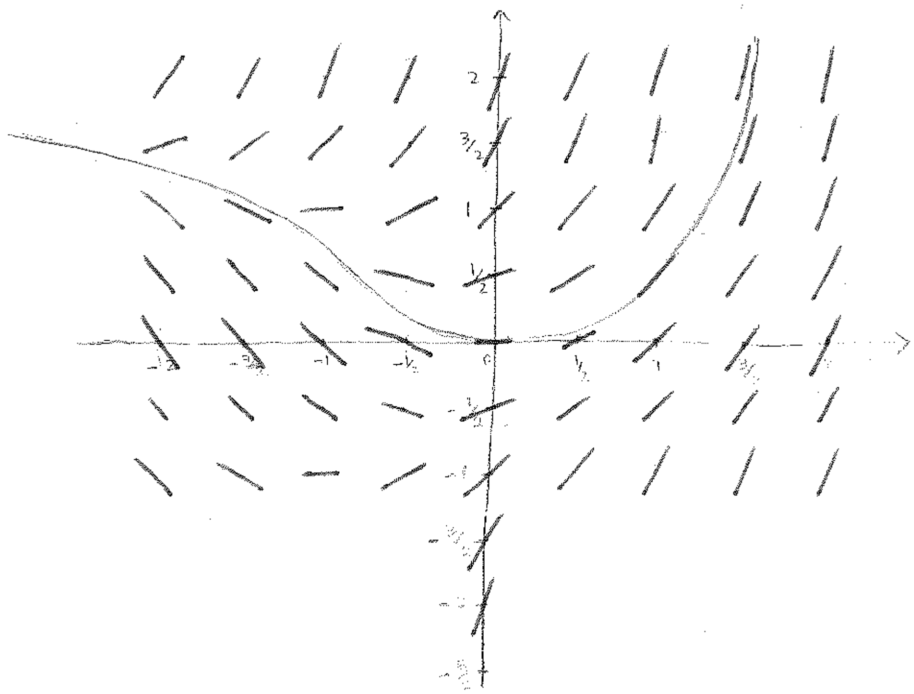
when $x<0, y>0$, $y'>0$ ✓

when $xy > \frac{1}{2}$ and $\lim_{x\to\infty} xy = \frac{1}{2}$ (for $x,y>0$ and $x>c>0$), then $y'<0$

for $x>c>0$ and $y>0$ and $\lim_{x\to\infty} y' = 0$ ✓

Section 9.2

14 $y' = x + y^2$



22 $y' = x^2 y - \frac{1}{2} y^2 \quad y(0) = 1$

$x_0 = 0 \ , \ y_0 = 1$

$x_1 = 0.2 \quad y_1 = y_0 + h(x_0^2 y_0 - \frac{1}{2} y_0^2) = 1 + 0.2(-\frac{1}{2} \cdot 1) = 0.9$

$x_2 = 0.4 \quad y_2 = y_1 + h(x_1^2 y_1 - \frac{1}{2} y_1^2) = 0.9 + 0.2((0.2)^2(0.9) - \frac{1}{2}(0.9)^2)$

$= 0.8262 \approx 0.83$

$x_3 = 0.6 \quad y_3 = y_2 + h(x_2^2 y_2 - \frac{1}{2} y_2^2) = 0.83 + 0.2((0.4)^2(0.83) - \frac{1}{2}(0.83)^2)$

$= 0.78767 \approx 0.79$

$x_4 = 0.8 \quad y_4 = y_3 + h(x_3^2 y_3 - \frac{1}{2} y_3^2) = 0.79 + 0.2((0.6)^2(0.79) - \frac{1}{2}(0.79)^2)$

$= 0.78447 \approx 0.78$

$x_5 = 1 \quad y_5 = y_4 + h(x_4^2 y_4 - \frac{1}{2} y_4^2) = 0.78 + 0.2((0.8)^2(0.78) - \frac{1}{2}(0.78)^2)$

$= 0.819 \approx 0.82$

$\boxed{\text{Section 9.3}}$

$\boxed{12}$ $\frac{dy}{dx} = \frac{x \sin x}{y}$: Separable equation

$$\to y\,dy = x \sin x\, dx \to \int y\,dy = \int x \sin x\, dx \underset{\substack{u = x \\ \sin x\, dx = dv \rightsquigarrow v = -\cos x}}{=} -x\cos x - \int -\cos x\, dx$$

$$\rightsquigarrow \frac{y^2}{2} = -x\cos x + \sin x + C$$

$$\rightsquigarrow y^2 = -2x\cos x + 2\sin x + 2C \rightsquigarrow y = \pm\sqrt{-2x\cos x + 2\sin x + 2C}$$

$$y(0) = \pm\sqrt{2C} = -1 \rightsquigarrow \sqrt{2C} = 1 \rightsquigarrow C = \frac{1}{2}$$

$$\to y = -\sqrt{-2x\cos x + 2\sin x + 1}$$

$\boxed{22}$ $xy' = y + x e^{y/x}$ $\quad v = \frac{y}{x} \rightsquigarrow y = xv \quad , y' = v + xv'$

$$\rightsquigarrow x(v + xv') = xv + x e^{v}$$

$$\to x^2 v' = x e^{v} \rightsquigarrow x v' = e^{v} \rightsquigarrow v' = \frac{e^{v}}{x} \quad \text{separable}$$

$$\rightsquigarrow \frac{dv}{e^{v}} = \frac{dx}{x} \rightsquigarrow \int \frac{dv}{e^{v}} = \int \frac{dx}{x} \rightsquigarrow -e^{-v} = \ln|x| + C$$

$$\to e^{-v} = -\ln|x| - C$$

$$\rightsquigarrow -v = \ln(-\ln|x| - C)$$

$$\rightsquigarrow -\frac{y}{x} = \ln(-\ln|x| - C)$$

$$\to y = -x\ln(-\ln|x| - C)$$

$\boxed{30}$ $y^2 = kx^3 \longrightarrow 2yy' = k(3x^2) \rightsquigarrow y' = \frac{3}{2}\frac{x^2}{y} \cdot k = \frac{3}{2}\frac{x^2}{y} \cdot \left(\frac{y^2}{x^3}\right)$

$$= \frac{3}{2} \cdot \frac{y}{x}$$

orthogonal trajectories $\rightsquigarrow$ are solutions of $\quad y' = -\frac{2}{3} \cdot \frac{x}{y} \rightsquigarrow \frac{dy}{dx} = -\frac{2}{3}\frac{x}{y}$ separable

$$\to y\,dy = -\frac{2}{3}x\,dx \rightsquigarrow \int y\,dy = \int -\frac{2}{3}x\,dx$$

$$\frac{y^2}{2} = -\frac{2}{3}\cdot\frac{x^2}{2} + C \rightsquigarrow \frac{y^2}{2} = -\frac{x^2}{3} + C \rightsquigarrow \boxed{3y^2 + 2x^2 = C}$$

$\boxed{32}$ $$y = \frac{1}{x+K} \rightsquigarrow \frac{dy}{dx} = -\frac{1}{(x+K)^2} \rightsquigarrow y' = -\frac{1}{\left(\frac{1}{y}\right)^2} = -y^2$$

$$x+K = \frac{1}{y} \rightsquigarrow K = \frac{1}{y} - x$$

$$\rightsquigarrow \frac{dy}{dx} = -y^2 \rightsquigarrow +\frac{dy}{y^2} = -dx \rightsquigarrow \int \frac{dy}{y^2} = \int -dx$$

$$\rightsquigarrow -\frac{1}{y} = -x + C$$

$$\rightsquigarrow \frac{1}{y} = x - C \rightsquigarrow \boxed{y = \frac{1}{x-C}}$$

$\boxed{34}$ $$y(x) = 2 + \int_1^x \frac{dt}{t\,y(t)} \qquad x > 0$$

1 - $y(1) = 2$

2 - $$y'(x) = \frac{1}{x y(x)} \rightsquigarrow \frac{dy}{dx} = \frac{1}{xy} \rightsquigarrow y\,dy = \frac{dx}{x}$$

$$\rightsquigarrow \int y\,dy = \int \frac{dx}{x} \rightsquigarrow \frac{y^2}{2} = \ln|x| + C \rightsquigarrow y^2 = \ln x^2 + 2C$$

$$\rightsquigarrow y = \pm\sqrt{\ln x^2 + 2C}$$

$$y(1) = 2 \rightsquigarrow y(1) = \sqrt{0 + 2C} = 2 \rightsquigarrow C = 2$$

$$\rightsquigarrow y = \sqrt{\ln x^2 + 4}$$

$\boxed{36}$ $$(t^2+1)f'(t) + (f(t))^2 + 1 = 0 \qquad t \neq 1 \qquad f(3) = 2$$

$$(t^2+1)f'(t) = -(f(t))^2 - 1$$

$$\rightsquigarrow -\frac{f'(t)}{(f(t))^2+1} = \frac{1}{t^2+1} \qquad u = f(t)$$

$$\rightsquigarrow \int -\frac{du}{u^2+1} = \int \frac{1}{t^2+1}\,dt \rightsquigarrow -\tan^{-1}u = \tan^{-1}t + C$$

$$\rightarrow u = \tan(-\tan^{-1}t - C) = \frac{-t - \tan C}{1 - t\cdot\tan C}$$

$$\rightarrow f(t) = \frac{-t - \tan C}{1 - t\cdot\tan C}, \quad f(3) = \frac{-3 - \tan C}{1 - 3\tan C} = 2 \Rightarrow \tan C = 1 \rightsquigarrow \frac{\pi}{4} = C, \boxed{f(t) = \frac{-t-1}{1-t} = \frac{t+1}{t-1}}$$