HOMEWORK 4

1. From [BT]: 4.11.1, 5.12* and 5.16*

2*. Let M be a smooth, compact and oriented n-manifold. The Euler characteristic of M is the number
$$\chi(M) = \sum_{p=0}^{n} (-1)^p \dim H^p_{dR}(M)$$
Show that $\chi(M)$ is a homotopy invariant of M, and $\chi(M) = 0$ when n is odd.

3*. (a) Read the statement of Künneth formula from section 5 of [BT] and compute the cohomology ring of $H^*_{dR}(T^n)$, where $T^n = \mathbb{R}^n/\mathbb{Z}^n$ denotes the n-dimensional torus.
   (b) Let A be an $n \times n$ matrix with integer entries. Then, A induces a $\phi : T^n \to T^n$. Show that $\phi^* : H^1_{dR}(T^n) \to H^1_{dR}(T^n)$ is given by the transpose of A.
   (c) Show that the degree of ϕ is the determinant of A.

References

[BT] Raoul Bott and Loring W. Tu, *Differential forms in algebraic topology*, Springer New York, NY.