Solutions-Problem set 1(section 1.1)

Saturday, January 30, 2016 9:49 PM

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$$\begin{vmatrix} x+2y+3z=0 \\ 4x+5y+6z=0 \\ 7x+8y+10z=0 \end{vmatrix} \xrightarrow[\text{③}-7\text{①}]{\text{②}-4\text{①}} \begin{vmatrix} x+2y+3z=0 \\ -3y-6z=0 \\ -6y-11z=0 \end{vmatrix}$$

$$\xrightarrow{-\frac{1}{3}\text{②}} \begin{vmatrix} x+2y+3z=0 \\ y+2z=0 \\ -6y-11z=0 \end{vmatrix} \xrightarrow[\text{③}+6\text{②}]{\text{①}-2\text{②}} \begin{vmatrix} x \quad -z=0 \\ y+2z=0 \\ z=0 \end{vmatrix}$$

$$\xrightarrow[\text{②}-2\text{③}]{\text{①}+\text{③}} \begin{vmatrix} x \quad =0 \\ y \quad =0 \\ z=0 \end{vmatrix} \rightsquigarrow \begin{cases} x=0 \\ y=0 \\ z=0 \end{cases}$$

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$$\begin{vmatrix} x+y-z=2 \\ x+2y+z=3 \\ x+y+(k^2-5)z=k \end{vmatrix} \xrightarrow[\text{③}-\text{①}]{\text{②}-\text{①}} \begin{vmatrix} x+y-z=2 \\ y+2z=1 \\ (k^2-4)z=k-2 \end{vmatrix}$$

$$\xrightarrow{\text{①}-\text{②}} \begin{vmatrix} x \quad -3z=1 \\ y+2z=1 \\ (k^2-4)z=k-2 \end{vmatrix}$$

If $k^2-4=0$, but $k-2\neq 0$ i.e. $\underline{k=-2}$ then: <u>No solution</u> (inconsistent)

If $\underline{k=2}$, then we have $\begin{vmatrix} x \quad -3z=1 \\ y+2z=1 \end{vmatrix}$ therefore:

$z=t$, $x=1+3t$, $y=1-2t$: <u>Infinitely many solutions</u>

If $k^2-4\neq 0$, then $\begin{vmatrix} x \quad -3z=1 \\ y+2z=1 \\ (k+2)z=1 \end{vmatrix} \xrightarrow{\frac{1}{k+2}\text{③}} \begin{vmatrix} x \quad -3z=1 \\ y+2z=1 \\ z=\frac{1}{k+2} \end{vmatrix}$

$$\xrightarrow[\text{①}+3\text{③}]{\text{②}-2\text{③}} \begin{vmatrix} x \quad =1+\frac{3}{k+2} \\ y \quad =1-\frac{2}{k+2} \\ z=\frac{1}{k+2} \end{vmatrix}$$ <u>Unique solution!</u>

36 $f(t)=a+bt+ct^2 \longrightarrow f'(t)=b+2ct$

$f(1)=1$ $\quad |\, a+b+c=11 \quad |\, a+b+c=11$

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$f(t) = a + bt + ct^2 \longrightarrow f'(t) = b + 2ct$

$$\begin{matrix} f(1)=1 \\ f(3)=3 \\ f'(2)=3 \end{matrix} \longrightarrow \left| \begin{matrix} a + b + c = 1 \\ a + 3b + 9c = 3 \\ b + 4c = 3 \end{matrix} \right| \xrightarrow[\text{②}-\text{①}]{} \left| \begin{matrix} a + b + c = 1 \\ 2b + 8c = 2 \\ b + 4c = 3 \end{matrix} \right|$$

$$\xrightarrow[\frac{1}{2}\text{②}]{} \left| \begin{matrix} a + b + c = 1 \\ b + 4c = 1 \\ b + 4c = 3 \end{matrix} \right| \xrightarrow[3-\text{②}]{\text{①}-\text{②}} \left| \begin{matrix} a \quad -3c = 0 \\ b + 4c = 1 \\ 0 = 2 \end{matrix} \right|$$

No solution!

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$$\left| \begin{matrix} x + 2y + 3z = a \\ 4x + 5y + 6z = b \\ 7x + 8y + 9z = c \end{matrix} \right| \xrightarrow[\text{③}-7\text{①}]{\text{②}-4\text{①}} \left| \begin{matrix} x + 2y + 3z = a \\ -3y - 6z = b - 4a \\ -6y - 12z = c - 7a \end{matrix} \right|$$

$$\xrightarrow{-\frac{1}{3}\text{②}} \left| \begin{matrix} x + 2y + 3z = a \\ y + 2z = -\frac{b}{3} + \frac{4}{3}a \\ -6y - 12z = c - 7a \end{matrix} \right| \xrightarrow[\text{③}+6\text{②}]{\text{①}-2\text{②}} \left| \begin{matrix} x \quad -z = -\frac{5}{3}a + \frac{2}{3}b \\ y + 2z = -\frac{b}{3} + \frac{4}{3}a \\ 0 = a - 2b + c \end{matrix} \right|$$

$\longrightarrow$ The system is consistent i.e. has at least one solution iff $a - 2b + c = 0$

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Line equation: $(x,y,z) = (1,1,1) + t\big(\overbrace{(3,5,0) - (1,1,1)}^{(2,4,-1)}\big) = (1+2t, 1+4t, 1-t)$

$$\longrightarrow t = \frac{x-1}{2} = \frac{y-1}{4} = 1-z \longrightarrow \begin{cases} \frac{x-1}{2} = 1-z \\ \frac{y-1}{4} = 1-z \end{cases} \longrightarrow \begin{cases} x + 2z = 3 \\ y + 4z = 5 \end{cases}$$