Solutions-Problem set 1(section 1.2)

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10:14 PM

$\boxed{8}$

$$\left|\begin{array}{l} x_2 + 2x_4 + 3x_5 = 0 \\ \qquad\;\; 4x_4 + 8x_5 = 0 \end{array}\right| \xrightarrow[\text{matrix}]{\text{Aug.}} \left[\begin{array}{ccccc:c} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 4 & 8 & & 0 \end{array}\right]$$

$$\xrightarrow[\frac{1}{4}②]{} \left[\begin{array}{ccccc:c} 0 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{array}\right] \xrightarrow[①-2②]{} \left[\begin{array}{ccccc:c} 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{array}\right]$$

$\longrightarrow$ x_1, x_3 and x_5 are free variables and $x_2 = x_5$ & $x_4 = -2x_5$

$$\longrightarrow \begin{cases} x_1 = t \\ x_2 = s \\ x_3 = r \\ x_4 = -2s \\ x_5 = s \end{cases}$$

$\boxed{12}$

$$\left|\begin{array}{l} 2x_1 \qquad\quad - 3x_3 \qquad\qquad + 7x_5 + 7x_6 = 0 \\ -2x_1 + x_2 + 6x_3 \qquad\quad - 6x_5 - 12x_6 = 0 \\ \qquad\quad x_2 - 3x_3 \qquad\quad + x_5 + 5x_6 = 0 \\ \qquad -2x_2 \qquad\quad + x_4 + x_5 + x_6 = 0 \\ 2x_1 + x_2 - 3x_3 \qquad\quad + 8x_5 + 7x_6 = 0 \end{array}\right|$$

$\Big\downarrow$ Augm. matrix

$$\begin{bmatrix} 2 & 0 & -3 & 0 & 7 & 7 & 0 \\ -2 & 1 & 6 & 0 & -6 & -12 & 0 \\ 0 & 1 & -3 & 0 & 1 & 5 & 0 \\ 0 & -2 & 0 & 1 & 1 & 1 & 0 \\ 2 & 1 & -3 & 0 & 8 & 7 & 0 \end{bmatrix}$$

$$\xrightarrow{⑤ - ① \;|\; ② + ①}$$

$$\left[\begin{array}{cccccc|c}
2 & 0 & -3 & 0 & 7 & 7 & 0 \\
0 & 1 & 3 & 0 & 1 & -5 & 0 \\
0 & 1 & -3 & 0 & 1 & 5 & 0 \\
0 & -2 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0
\end{array}\right]$$

$$\xrightarrow{\frac{1}{2}①}$$

$$\left[\begin{array}{cccccc|c}
1 & 0 & -\frac{3}{2} & 0 & \frac{7}{2} & \frac{7}{2} & 0 \\
0 & 1 & 3 & 0 & 1 & -5 & 0 \\
0 & 1 & -3 & 0 & 1 & 5 & 0 \\
0 & -2 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0
\end{array}\right]$$

$$\xrightarrow{④ + 2② \;|\; ③ - ② \;|\; ⑤ - ②}$$

$$\left[\begin{array}{cccccc|c}
1 & 0 & -\frac{3}{2} & 0 & \frac{7}{2} & \frac{7}{2} & 0 \\
0 & 1 & 3 & 0 & 1 & -5 & 0 \\
0 & 0 & -6 & 0 & 0 & 10 & 0 \\
0 & 0 & 6 & 1 & 3 & -9 & 0 \\
0 & 0 & -3 & 0 & 0 & 5 & 0
\end{array}\right]$$

$$\xrightarrow{-\frac{1}{6}③}$$

$$\left[\begin{array}{cccccc|c}
1 & 0 & -\frac{3}{2} & 0 & \frac{7}{2} & \frac{7}{2} & 0 \\
0 & 1 & 3 & 0 & 1 & -5 & 0 \\
0 & 0 & 1 & 0 & 0 & -\frac{5}{3} & 0 \\
0 & 0 & 6 & 1 & 3 & -9 & 0 \\
0 & 0 & -3 & 0 & 0 & 5 & 0
\end{array}\right]$$

$$\xrightarrow{① + \frac{3}{2}③ \;|\; ④ - 6③ \;|\; ② - 3③ \;|\; ⑤ + 3③}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 7/2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -5/3 & 0 \\ 0 & 0 & 0 & 1 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{cases} x_1 = -\frac{7}{2}t - r \\ x_2 = -t \\ x_3 = \frac{5}{3}r \\ x_4 = -3t - r \\ x_5 = t \\ x_6 = r \end{cases}$$

$\boxed{18}$ (a)

$$\begin{array}{c} \begin{matrix} 1 & 2 & 0 & 2 & 0 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

This is a leading 1, but there is a nonzero entry in its Column.

$\Rightarrow$ It's not in reduced row echelon form.

(b) $\begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ ✓ It's in rref.

(c) $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$: Not ordered!

(The zero row should be in the bottom)

$\to$ Not in rref.

(d) $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}$ ✓ It's in rref

$\boxed{20}$ $\begin{bmatrix} 0 & a & 2 & 1 & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & e & 0 & 0 \end{bmatrix}$

- 1st row should have a leading 1.
  $\rightsquigarrow$ $a = 1$
- Since 4th Column, has 1 in its first row $\rightsquigarrow$ c can't be a leading 1 $\rightsquigarrow$ $c = 0$
  Similarly, $e = 0$
  (it also should be zero, because of order on the leading 1s)
- If $b = 0$, then $d = 0$ or 1.

- If $b \neq 0$, then $d=0$

$\Rightarrow$ Therefore, $a=1,\ b=0,\ c=0,\ e=0,\ d=0$

$a=1,\ b=0,\ c=0,\ e=0,\ d=1$

$a=1,\ b=t\neq 0,\ c=0,\ e=0,\ d=0$

$\boxed{37}$

$$\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = 0 \rightsquigarrow x_1+x_2+x_3+x_4=0$$

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix} \cdot \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = 0 \rightsquigarrow x_1+2x_2+3x_3+4x_4=0$$

$$\begin{bmatrix}1\\9\\9\\7\end{bmatrix} \cdot \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = 0 \rightsquigarrow x_1+9x_2+9x_3+7x_4=0$$

$$\rightsquigarrow \begin{cases} x_1+x_2+x_3+x_4=0 \\ x_1+2x_2+3x_3+4x_4=0 \\ x_1+9x_2+9x_3+7x_4=0 \end{cases} \xrightarrow[\text{matrix}]{\text{Aug}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 1 & 9 & 9 & 7 & 0 \end{array}\right]$$

$$\xrightarrow[(3)-(1)]{(2)-(1)} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 8 & 8 & 6 & 0 \end{array}\right] \xrightarrow[(3)-8(2)]{(1)-(2)} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -2 & 0 \\ & 1 & 2 & 3 & 0 \\ 0 & 0 & -8 & -18 & 0 \end{array}\right]$$

$$\xrightarrow[-\frac{1}{8}(3)]{} \left[\begin{array}{cccc|c} 1 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 9/4 & 0 \end{array}\right] \xrightarrow[(2)-2(3)]{(1)+(3)} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/4 & 0 \\ 0 & 1 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 9/4 & 0 \end{array}\right]$$

$$\longrightarrow \begin{cases} x_1 = -\frac{1}{4}t \\ x_2 = \frac{3}{2}t \\ x_3 = -\frac{9}{4}t \\ x_4 = t \end{cases}$$

$\boxed{48}$

$$\left|\begin{matrix} y+2kz=0 \\ x+2y+6z=2 \\ kx \quad +2z=1 \end{matrix}\right| \longrightarrow \left[\begin{array}{ccc|c} 0 & 1 & 2k & 0 \\ 1 & 2 & 6 & 2 \\ k & 0 & 2 & 1 \end{array}\right]$$

$$\xrightarrow{②-2①} \left[\begin{array}{ccc|c} 0 & 1 & 2k & 0 \\ 1 & 0 & 6-4k & 2 \\ k & 0 & 2 & 1 \end{array}\right] \xrightarrow{③-k②} \left[\begin{array}{ccc|c} 0 & 1 & 2k & 0 \\ 1 & 0 & 6-4k & 2 \\ 0 & 0 & 2-6k+4k^2 & 1-2k \end{array}\right]$$

$2-6k+4k^2=(2k-1)(2k-2)$

⇝ If $k=1$, then $2-6k+4k^2=0$ but $1-2k\neq 0$, therefore No solution.
If $k=\frac{1}{2}$, then both $2-6k+4k^2$ and $1-2k$ vanish, thus x_3 is a free variable and system has infinitely many solutions.

If $k\neq 1, \frac{1}{2}$, then $2-6k+4k^2\neq 0$, thus

$$\xrightarrow[\frac{1}{2-6k+4k^2}\times ③]{} \left[\begin{array}{ccc|c} 0 & 1 & 2k & 0 \\ 1 & 0 & 6-4k & 2 \\ 0 & 0 & 1 & \frac{1}{2-2k} \end{array}\right]$$

$$\xrightarrow[②-(6-4k)③]{①-2k③} \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2-\frac{6-4k}{2-2k} \\ 0 & 0 & 1 & \frac{1}{2-2k} \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2-2k} \end{array}\right] \longrightarrow$$

One solution

$x_1=\frac{1}{k-1}$

$x_2=0$

$x_3=\frac{1}{2-2k}$