Solutions-Problem set 1(section 1.3)

Sunday, January 31, 2016 3:20 PM

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a. System is inconsistent, because the 3rd row is $[0 \ 0 \ 0 \ \vdots \ 1]$. Therefore, <u>No solution</u>.

b. System is Consistent with no free variables, thus <u>One Solution</u>

c. System is Consistent, with one free variable "x_3", thus, <u>infinitely solutions</u>.

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$$\begin{bmatrix}1&4&7\\2&5&8\\3&6&9\end{bmatrix} \xrightarrow[③-3①]{②-2①} \begin{bmatrix}1&4&7\\0&-3&-6\\0&-6&-12\end{bmatrix} \xrightarrow[-\frac{1}{3}②]{} \begin{bmatrix}1&4&7\\0&1&2\\0&-6&-12\end{bmatrix}$$

$$\xrightarrow[③+6②]{①-4②} \begin{bmatrix}1&0&-1\\0&1&2\\0&0&0\end{bmatrix}$$ $\rightsquigarrow$ in reduced row echelon form

$\rightsquigarrow$ rank = # (leading 1s)

$= 2$

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In terms of columns:

$$\begin{bmatrix}1&2&3\\2&3&4\end{bmatrix}\begin{bmatrix}-1\\2\\1\end{bmatrix} = -1\begin{bmatrix}1\\2\end{bmatrix} + 2\begin{bmatrix}2\\3\end{bmatrix} + 1\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}6\\8\end{bmatrix}$$

In terms of rows:

$$\begin{bmatrix}1&2&3\\2&3&4\end{bmatrix}\begin{bmatrix}-1\\2\\1\end{bmatrix} = \begin{bmatrix}1\cdot(-1)+2\cdot2+3\cdot1\\2\cdot(-1)+3\cdot2+4\cdot1\end{bmatrix} = \begin{bmatrix}6\\8\end{bmatrix}$$

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$A\vec{x} = \vec{b}$ has a unique solution, therefore:

$$\text{rref}(A) = \begin{bmatrix}1&0&\cdots&\cdots&0\\0&1&0&\cdots&0\\\vdots&&1&&\vdots\\&&&\ddots&\\0&0&\cdots&&1\end{bmatrix} = I_n \quad (\text{rank}(A)=n)$$

(A is the Coff. matrix of linear system $\rightsquigarrow A\vec{x} = \vec{c}$ has exactly one solution.

$\boxed{34}$

a. $A\vec{e}_1 = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\begin{bmatrix} a \\ d \\ g \end{bmatrix} + 0\begin{bmatrix} b \\ e \\ h \end{bmatrix} + 0\begin{bmatrix} c \\ f \\ k \end{bmatrix} = \begin{bmatrix} a \\ d \\ g \end{bmatrix}$

$$A\vec{e}_2 = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ e \\ h \end{bmatrix}$$

$$A\vec{e}_3 = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} c \\ f \\ k \end{bmatrix}$$

b.

$$B\vec{e}_1 = \begin{bmatrix} | & | & | \\ \vec{V}_1 & \vec{V}_2 & \vec{V}_3 \\ | & | & | \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 1\cdot\vec{V}_1 + 0\vec{V}_2 + 0\cdot\vec{V}_3 = \vec{V}_1$$

$$B\vec{e}_2 = \begin{bmatrix} | & | & | \\ \vec{V}_1 & \vec{V}_2 & \vec{V}_3 \\ | & | & | \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0\cdot\vec{V}_1 + 1\cdot\vec{V}_2 + 0\cdot\vec{V}_3 = \vec{V}_2$$

$$B\vec{e}_3 = \begin{bmatrix} | & | & | \\ \vec{V}_1 & \vec{V}_2 & \vec{V}_3 \\ | & | & | \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0\cdot\vec{V}_1 + 0\cdot\vec{V}_2 + 1\cdot\vec{V}_3 = \vec{V}_3$$

$\boxed{36}$ Following question 34

$A\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \rightsquigarrow$ 1st Column of $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$A\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \rightsquigarrow$ 2nd Column of $A = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$A\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \rightsquigarrow$ 3rd Column of $A = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$

$$\Longrightarrow A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

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a. $A\vec{x}=\vec{0} \rightsquigarrow$ Aug. matrix $= \left[\begin{array}{c|c} & 0 \\ A & \vdots \\ & 0 \end{array}\right]$

Apply elementary row operations doesn't change the last Column of the augmented matrix, so

$$\text{rref (Aug. matrix)} = \left[\begin{array}{c|c} & 0 \\ \text{rref}(A) & \vdots \\ & 0 \end{array}\right]$$

Thus it has no row of the form $[0\ 0 \cdots 0 \vdots 1]$.

$\Rightarrow$ Consistent.

b. Any homo. system is Consistent, So it has at least one solution.
If # equ. < # unknown, then it can't have one solution.
Thus it has infinitely many solutions.

Or, you can say # equs < # unknowns

$\Downarrow$

rows of A < # Columns of A

Since rank (A) $\leq$ # rows of A < # Columns of A

$\Rightarrow$ rref(A) has Columns with no leading 1.

$\Rightarrow$ System has free variable(s)

$\Rightarrow$ Since it's Consistent, it has infinitely solutions.

c. $A(\vec{x_1}+\vec{x_2}) = A\vec{x_1} + A\vec{x_2} = 0$

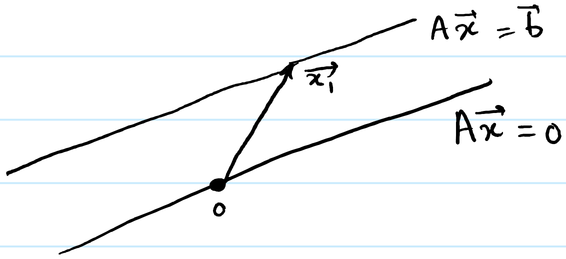
$\rightarrow \vec{x_1}+\vec{x_2}$ is a solution.

d. $A(k\vec{x}) = kA\vec{x} = 0 \rightsquigarrow k\vec{x}$ is a solution.

$\boxed{48}$ a. $A(\vec{x}_1+\vec{x}_h) = A\vec{x}_1 + A\vec{x}_h = \vec{b}+\vec{0} = \vec{b}$

$A(\vec{x}_2-\vec{x}_1) = A\vec{x}_2 + A(-\vec{x}_1) = A\vec{x}_2 - A\vec{x}_1 = \vec{b}-\vec{b} = 0$

c. solutions of $A\vec{x}=\vec{b}$ are of the form $\vec{x}_1+\vec{x}_h$ where $\vec{x}_h$ is a solution of $A\vec{x}=0$. Therefore: it's a line parallel to L, passing $\vec{x}_1$.



$\boxed{60}$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = x_1 \begin{bmatrix} 0 \\ 0 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 3 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ when this system has a solution?

$$\Rightarrow \left[\begin{array}{ccc|c} 0 & 1 & 2 & a \\ 0 & 0 & 0 & b \\ 3 & 4 & 5 & c \\ 0 & 0 & 6 & d \end{array}\right] \xrightarrow{(3)-4(1)} \left[\begin{array}{ccc|c} 0 & 1 & 2 & a \\ 0 & 0 & 0 & b \\ 3 & 0 & -3 & c-4a \\ 0 & 0 & 6 & d \end{array}\right]$$

$$\xrightarrow{\frac{1}{3}(3)} \left[\begin{array}{ccc|c} 0 & 1 & 2 & a \\ 0 & 0 & 0 & b \\ 1 & 0 & -1 & (c-4a)/3 \\ 0 & 0 & 6 & d \end{array}\right] \xrightarrow{\frac{1}{6}\times(4)} \left[\begin{array}{ccc|c} 0 & 1 & 2 & a \\ 0 & 0 & 0 & b \\ 1 & 0 & -1 & (c-4a)/3 \\ 0 & 0 & 1 & d/6 \end{array}\right]$$

$$\xrightarrow[(3)+(4)]{(1)-2(4)} \left[\begin{array}{ccc|c} 0 & 1 & 0 & a-d/3 \\ 0 & 0 & 0 & b \\ 1 & 0 & 0 & (c-4a)/3+d/6 \\ 0 & 0 & 1 & d/6 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{c-4a}{3}+\frac{d}{6} \\ 0 & 1 & 0 & a-\frac{d}{3} \\ 0 & 0 & 1 & \frac{d}{6} \\ 0 & 0 & 0 & b \end{array}\right]$$

$\Rightarrow b=0$ and a, c, d can take any value.