Solutions-Problem set 2(section 2.1)

Saturday, February 6, 2016 9:13 PM

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$$T\begin{bmatrix}x_1\\x_2\end{bmatrix}=x_1\begin{bmatrix}1\\2\\3\end{bmatrix}+x_2\begin{bmatrix}4\\5\\6\end{bmatrix}=\begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} \rightsquigarrow T \text{ is linear and its matrix is } \begin{bmatrix}1&4\\2&5\\3&6\end{bmatrix}$$

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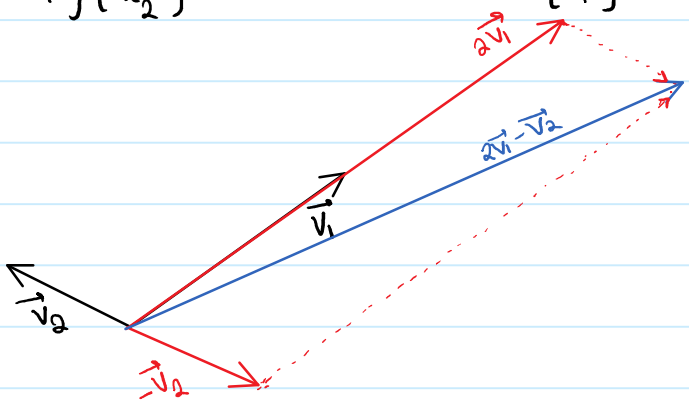
$$T(\vec{x})=A\vec{x}=\begin{bmatrix}\vec{v}_1&\vec{v}_2\\1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=x_1\vec{v}_1+x_2\vec{v}_2 \rightsquigarrow T\begin{bmatrix}2\\-1\end{bmatrix}=2\vec{v}_1-\vec{v}_2$$



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$$\vec{v}=\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix} \qquad T(\vec{x})=\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}\times\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}v_2x_3-v_3x_2\\v_3x_1-v_1x_3\\v_1x_2-v_2x_1\end{bmatrix}=\begin{bmatrix}-v_3x_2+v_2x_3\\v_3x_1-v_1x_3\\-v_2x_1+v_1x_2\end{bmatrix}$$

$$=\underbrace{\begin{bmatrix}0&-v_3&v_2\\v_3&0&-v_1\\-v_2&v_1&0\end{bmatrix}}_{\text{matrix}}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \quad : \underline{\text{Linear}}$$

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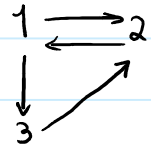


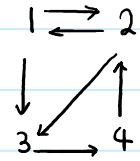
$\rightsquigarrow$ transition matrix: $\begin{bmatrix}0&1&0\\\frac{1}{2}&0&1\\\frac{1}{2}&0&0\end{bmatrix}=A$

$$A\vec{x}=\vec{x} \rightsquigarrow \begin{bmatrix}0&1&0\\\frac{1}{2}&0&1\\\frac{1}{2}&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \rightsquigarrow \begin{bmatrix}-1&1&0\\\frac{1}{2}&-1&1\\\frac{1}{2}&0&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=0$$

$$\rightsquigarrow \left[\begin{array}{ccc|c}-1&1&0&0\\\frac{1}{2}&-1&1&0\\\frac{1}{2}&0&-1&0\end{array}\right] \xrightarrow[\frac{1}{2}①+③]{\frac{1}{2}①+②} \left[\begin{array}{ccc|c}-1&1&0&0\\0&-\frac{1}{2}&1&0\\0&\frac{1}{2}&-1&0\end{array}\right] \xrightarrow[2②+①]{②+③} \left[\begin{array}{ccc|c}-1&0&2&0\\0&-\frac{1}{2}&1&0\\0&0&0&0\end{array}\right]$$

$$\xrightarrow[-2②]{-1①} \left[\begin{array}{ccc|c}1&0&-2&0\\0&1&-2&0\\0&0&0&0\end{array}\right] \Rightarrow \begin{cases}x_1=2t\\x_2=2t\\x_3=t\end{cases} \quad 2t+2t+t=1 \rightsquigarrow t=\frac{1}{5} \quad \vec{x}_{eq}=\begin{bmatrix}2/5\\2/5\\1/5\end{bmatrix}$$

$\Rightarrow$ Pages 2 & 3

53 a. 

$$B=\begin{bmatrix} 0+0.05 & 0.4+0.05 & 0+0.05 & 0+0.05 \\ 0.4+0.05 & 0+0.05 & 0+0.05 & 0.8+0.05 \\ 0.4+0.05 & 0.4+0.05 & 0+0.05 & 0+0.05 \\ 0+0.05 & 0+0.05 & 0.8+0.05 & 0+0.05 \end{bmatrix}$$

$$=\begin{bmatrix} 0.05 & 0.45 & 0.05 & 0.05 \\ 0.45 & 0.05 & 0.05 & 0.85 \\ 0.45 & 0.45 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.85 & 0.05 \end{bmatrix}$$

b. For c_j = # links going out of the page j

$\rightsquigarrow a_{ij}=\frac{1}{c_j}$ if $i \bullet \longleftarrow \bullet j$ there is a link from j to i otherwise $a_{ij}=0$

In the new model, $b_{ij}=\underbrace{\frac{80}{100}\cdot\frac{1}{c_j}}_{\text{80\% randomly follow a link}}+\underbrace{\frac{20}{100}\cdot\frac{1}{N}}_{\text{20\% randomly jump}}$: if $i \bullet \xleftarrow{\exists} \bullet j$

$b_{ij}=\frac{20}{100}\cdot\frac{1}{N}$ if no link

$\rightsquigarrow b_{ij}=\frac{80}{100}a_{ij}+\frac{20}{100}\cdot\frac{1}{N} \Rightarrow B=0.8A+\frac{0.2}{N}\cdot E$ (all entries equal "1")

c. $B\vec{x}_{equ}=\vec{x}_{equ} \rightsquigarrow \begin{bmatrix} -\frac{19}{20} & \frac{9}{20} & \frac{1}{20} & \frac{1}{20} \\ \frac{9}{20} & -\frac{19}{20} & \frac{1}{20} & \frac{17}{20} \\ \frac{9}{20} & \frac{9}{20} & -\frac{19}{20} & \frac{1}{20} \\ \frac{1}{20} & \frac{1}{20} & \frac{17}{20} & -\frac{19}{20} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\vec{0}$

$\rightsquigarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} t \\ \frac{9}{5}t \\ \frac{7}{5}t \\ \frac{7}{5}t \end{bmatrix}$

$\rightsquigarrow \underbrace{\left(1+\frac{9}{5}+\frac{7}{5}+\frac{7}{5}\right)}_{\frac{28}{5}}t=1 \rightsquigarrow t=\frac{5}{28}$

$\rightarrow \vec{x}_{equ}=\begin{bmatrix} 5/28 \\ 9/28 \\ 1/4 \\ 1/4 \end{bmatrix}$

56 a. $B = 0.8A + \frac{0.2}{N}E$

$$\rightsquigarrow B = 0.8\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \end{bmatrix} + \frac{0.2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{15} & \frac{13}{15} & \frac{1}{15} \\ \frac{7}{15} & \frac{1}{15} & \frac{13}{15} \\ \frac{7}{15} & \frac{1}{15} & \frac{1}{15} \end{bmatrix}$$

b. $B\vec{x} = \vec{x} \rightsquigarrow \begin{bmatrix} -\frac{14}{15} & \frac{13}{15} & \frac{1}{15} \\ \frac{7}{15} & -\frac{14}{15} & \frac{13}{15} \\ \frac{7}{15} & \frac{1}{15} & -\frac{14}{15} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0}$

$$\rightsquigarrow \vec{x} = \begin{bmatrix} t \\ \frac{63}{61}t \\ \frac{35}{61}t \end{bmatrix} \rightsquigarrow \frac{159}{61}t = 1 \rightsquigarrow t = \frac{61}{159} \Rightarrow \begin{bmatrix} \frac{61}{159} \\ \frac{63}{159} \\ \frac{35}{159} \end{bmatrix} = \vec{x}_{equ.}$$

c. Page 2