Solutions-Problem set2(section 2.2)

Saturday, February 6, 2016 9:41 PM

6 $\vec{w} = \begin{bmatrix}2\\1\\2\end{bmatrix}$ $\vec{x} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ $\vec{x}_{\parallel} = \frac{\vec{x}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\vec{w} = \frac{1}{\begin{bmatrix}2\\1\\2\end{bmatrix}\cdot\begin{bmatrix}2\\1\\2\end{bmatrix}}\left(\begin{bmatrix}2\\1\\2\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\end{bmatrix}\right)\begin{bmatrix}2\\1\\2\end{bmatrix}$

$= \frac{5}{9}\begin{bmatrix}2\\1\\2\end{bmatrix} = \begin{bmatrix}10/9\\5/9\\10/9\end{bmatrix}$

12 A : reflection $\rightsquigarrow A = \begin{bmatrix}a & b\\ b & -a\end{bmatrix}$ and $a^2+b^2=1$

(a) $A(A\begin{bmatrix}x_1\\x_2\end{bmatrix}) = A\left(\underbrace{\begin{bmatrix}a & b\\ b & -a\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}}_{\begin{bmatrix}ax_1+bx_2\\ bx_1-ax_2\end{bmatrix}}\right) = \begin{bmatrix}a & b\\ b & -a\end{bmatrix}\begin{bmatrix}ax_1+bx_2\\ bx_1-ax_2\end{bmatrix}$

$= \begin{bmatrix}a(ax_1+bx_2)+b(bx_1-ax_2)\\ b(ax_1+bx_2)-a(bx_1-ax_2)\end{bmatrix} \overset{a^2+b^2=1}{=\!=} \begin{bmatrix}x_1\\x_2\end{bmatrix}$

(b) $\vec{v} = \vec{x} + A\vec{x} \rightsquigarrow A\vec{v} = A(\vec{x}+A\vec{x}) = A\vec{x} + A(A\vec{x}) = A\vec{x} + \vec{x} = \vec{v}$ ($A(A\vec{x}) = \vec{x}$)

$\rightsquigarrow \boxed{A\vec{v} = \vec{v}}$

(c) $\vec{w} = \vec{x} - A\vec{x} \rightsquigarrow A\vec{w} = A(\vec{x}-A\vec{x}) = A\vec{x} - \underbrace{A(A\vec{x})}_{\vec{x}} = A\vec{x} - \vec{x} = -\vec{w}$

$\rightsquigarrow \boxed{A\vec{w} = -\vec{w}}$

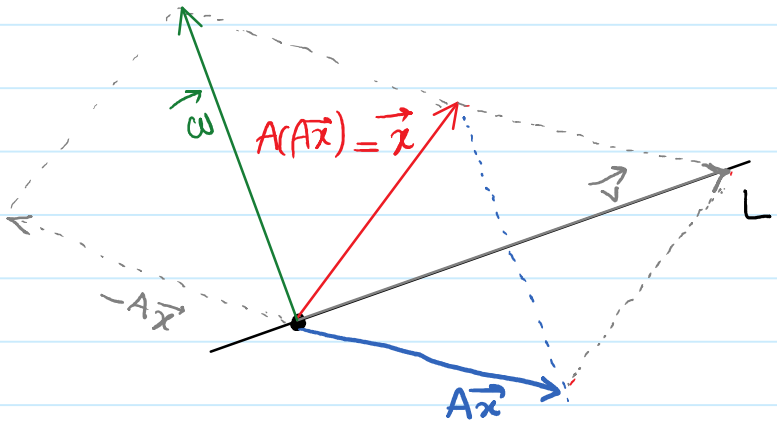
(d) $\vec{v}\cdot\vec{w} = (\vec{x}+A\vec{x})\cdot(\vec{x}-A\vec{x}) = \vec{x}\cdot\vec{x} + (A\vec{x})\cdot\vec{x} - \vec{x}\cdot A\vec{x} - (A\vec{x})\cdot(A\vec{x})$

$= \vec{x}\cdot\vec{x} - (A\vec{x})\cdot(A\vec{x}) = 0$

$(A\vec{x})\cdot\vec{x} = \vec{x}\cdot(A\vec{x})$; $|x| = |A\vec{x}|$

$\vec{v}$ is perp. to $\vec{w}$ $\rightsquigarrow \pi/2$

(e) $A\vec{v} = \vec{v} \rightsquigarrow \vec{v}$ lies on line L.



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a. $\begin{bmatrix} 8 \\ -4 \end{bmatrix} = 4\begin{bmatrix} 2 \\ -1 \end{bmatrix} \rightsquigarrow T(\vec{x}) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\vec{x}$

b. $\begin{bmatrix} u_1^2 & u_1u_2 \\ u_1u_2 & u_2^2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \rightsquigarrow \begin{cases} 2u_1^2 + 3u_1u_2 = 2 \\ 2u_1u_2 + 3u_2^2 = 0 \end{cases} \Rightarrow \begin{cases} 2(1-u_2^2) + 3u_1u_2 = 2 \\ 2u_1u_2 + 3u_2^2 = 0 \end{cases}$

$u_1^2 + u_2^2 = 1$

$\Rightarrow \begin{cases} -2u_2^2 + 3u_1u_2 = 0 \\ 2u_1u_2 + 3u_2^2 = 0 \end{cases} \rightsquigarrow \begin{cases} -2u_2^2 + 3u_1u_2 = 0 \\ u_1u_2 = 0 \end{cases} \Rightarrow u_2^2 = 0$

$\Rightarrow u_2 = 0 \longrightarrow u_1 = \pm 1 \rightsquigarrow T(\vec{x}) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

c. $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \rightsquigarrow \begin{cases} -5b = 3 \\ 5a = 4 \end{cases} \rightsquigarrow a = \frac{4}{5}, b = -\frac{3}{5}$

$\rightsquigarrow T(\vec{x}) = \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{bmatrix}\vec{x}$

d. $\begin{bmatrix} 1 \\ 3 \end{bmatrix} \longrightarrow \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ horizontal shear $\rightsquigarrow \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$\rightsquigarrow 1 + 3k = 7 \rightsquigarrow k = 2 \rightsquigarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

e. $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}\begin{bmatrix} 7 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 5 \end{bmatrix} \rightsquigarrow \begin{cases} 7a + b = -5 \\ -a + 7b = 5 \end{cases} \rightsquigarrow \begin{cases} 7a + b = -5 \\ -50a = 40 \end{cases}$

$\rightsquigarrow \begin{cases} 7a + b = -5 \\ a = -\frac{4}{5} \end{cases} \rightsquigarrow \begin{cases} b = \frac{3}{5} \\ a = -\frac{4}{5} \end{cases} \rightsquigarrow \begin{bmatrix} -\frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix}$

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a. Scaling ..... $D = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

b. Shear ..... $E = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ (vertical shear)

c. Rotation ..... $C = \begin{bmatrix} -0.6 & 0.8 \\ -0.8 & -0.6 \end{bmatrix}$ $(a = -0.6,\ b = -0.8)$

d. Orthogonal proj .... $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $(u_1 = 0,\ u_2 = 1)$

e. Reflection ..... $F = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$ $(a = 0.6 \quad b = 0.8)$