Solutions-Problem set 2(section 2.3)

Saturday, February 6, 2016 10:45 PM

8 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

14 $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B=\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ $C=\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ $D=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $E=\begin{bmatrix} 5 \end{bmatrix}$

Size: 2×2 1×3 3×3 3×1 1×1

Defined multiplications: AA

BC, BD

CC, CD

DB, DE

EB, EE

$AA=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ $BC=\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}=\begin{bmatrix} 14 & 8 & 2 \end{bmatrix}$

$BD=\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 6 \end{bmatrix}$ $CC=\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}=\begin{bmatrix} -2 & -2 & -2 \\ 4 & 1 & -2 \\ 10 & 4 & -2 \end{bmatrix}$

$CD=\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}$ $DB=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}=\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

$DE=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\begin{bmatrix} 5 \end{bmatrix}=\begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$ $EB=\begin{bmatrix} 5 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}=\begin{bmatrix} 5 & 10 & 15 \end{bmatrix}$

$EE=\begin{bmatrix} 5 \end{bmatrix}\begin{bmatrix} 5 \end{bmatrix}=\begin{bmatrix} 25 \end{bmatrix}$

24 $A=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $B=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$

$AB=\begin{bmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 3g & 3h & 3k \end{bmatrix}$ $BA=\begin{bmatrix} 2a & 2b & 3c \\ 2d & 2e & 3f \\ 2g & 2h & 3k \end{bmatrix}$

$AB=BA \Longrightarrow \begin{cases} 3g=2g \\ 3h=2h \\ 3c=2c \\ 3f=2f \end{cases} \longrightarrow g=h=c=f \rightsquigarrow B=\begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & k \end{bmatrix}$

$\boxed{40}$ $A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$ $A^2=AA=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}0&-1\\1&0\end{bmatrix}=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}=-I_2 \Rightarrow A^3=A\underbrace{A^2}_{-I_2}=-A=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$

$A^4=\underbrace{A^2}_{-I_2}\underbrace{A^2}_{-I_2}=I_2$ Since $A^4=I_2$, $A^{4K}=I_2$

$A^{4k+1}=A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

$A^{4k+2}=-I_2$

$A^{4k+3}=-A=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$

$1001=4(250)+1 \leadsto A^{1001}=A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

$A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}=\begin{bmatrix}\cos\frac{\pi}{2}&-\sin\frac{\pi}{2}\\ \sin\frac{\pi}{2}&\cos\frac{\pi}{2}\end{bmatrix}$: 90° rotation

$\Rightarrow$ A^2: rotation with angle π i.e. $A^2\vec{x}=-\vec{x} \leadsto A^2=-I_2$

A^3: rotation with angle $\frac{3\pi}{2} \leadsto A^3=-A$

A^4: " " " $2\pi \leadsto A^4=I_2$

for any $n=4k+1$, A^{4k+1} is rotation with angle $(4k+1)\frac{\pi}{2}=2\pi+\frac{\pi}{2}$

$\leadsto$ it's equal to rotation of angle $\frac{\pi}{2} \leadsto A^{4k+1}=A$.

(Similarly for $4k+3$, $4k+2$ & $4k$)

$\boxed{44}$ $A^2\neq I_2$, $A^4=I_2$ (look at exercise 40)

For $T=\pi/2$ rotation, $T^2=-Id$ & $T^4=Id \leadsto A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$