Solutions-Problem set 4(section 3.2)

Wednesday, March 2, 2016 10:15 AM

$\boxed{32}$

$$A=\begin{bmatrix}0&1&2&0&0&3\\0&0&0&1&0&4\\0&0&0&0&1&5\\0&0&0&0&0&0\end{bmatrix} \rightsquigarrow im(A)=\mathrm{Span}\left(\begin{bmatrix}0\\0\\0\\0\end{bmatrix},\underbrace{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}},\begin{bmatrix}2\\0\\0\\0\end{bmatrix},\underbrace{\begin{bmatrix}0\\1\\0\\0\end{bmatrix}},\underbrace{\begin{bmatrix}0\\0\\1\\0\end{bmatrix}},\begin{bmatrix}3\\4\\5\\0\end{bmatrix}\right)$$

redundant

$$\to\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\right): \text{basis for } im(A)$$

$\boxed{42}$ Let $c_1,\dots,c_m$ in $\mathbb{R}$ be a solution of

$$c_1\vec{v}_1+\dots+c_m\vec{v}_m=\vec{0}\longleftrightarrow \underbrace{\begin{bmatrix}|&|&&|\\\vec{v}_1&\vec{v}_2&\cdots&\vec{v}_m\\|&|&&|\end{bmatrix}}_{A}\begin{bmatrix}c_1\\c_2\\\vdots\\c_m\end{bmatrix}=\vec{0}$$

i.e. $\begin{bmatrix}c_1\\\vdots\\c_m\end{bmatrix}\in \ker(A)$

Then $\vec{v}_i\cdot(c_1\vec{v}_1+\dots+c_m\vec{v}_m)=0$

$\Rightarrow c_1(\vec{v}_i\cdot\vec{v}_1)+\dots+c_m(\vec{v}_i\cdot\vec{v}_m)=0$

$\vec{v}_i\cdot\vec{v}_j=0$ for any $j\neq i$ $\rightsquigarrow c_i(\overset{1}{\overset{\shortparallel}{\vec{v}_i\cdot\vec{v}_i}})=0$

$\rightsquigarrow c_i=0$

$\Rightarrow c_1=c_2=\dots=c_m=0 \rightsquigarrow \vec{v}_1,\dots,\vec{v}_m$ are linearly independent