Homework 4

Due: Feb 21

1. Recall that a triple (E, B, π) is a fibration, if it has the homotopy lifting property with respect to $(X, \emptyset)$ for all topological spaces X.

   (a) Show that a fibration is a Serre fibration.

   (b) Show that all fibers of a fibration over a path connected space B are homotopy equivalent.

   (c) Show that for any Serre fibration (E, B, π), $\{H_\star(F_x)\}$ is a local coefficient system on B.

2. Give an example of a filtered complex of vector spaces, whose associated spectral sequence degenerated at (a) E^1, (b) E^2, (c) E^n.

3. Write all pages of the spectral sequence for the Hopf fibration $\mathbb{S}^1 \to \mathbb{S}^{2n+1} \to \mathbb{CP}^n$. At what page does the sequence degenerate?

4. Read example 1.6 of Hatcher's book "Spectral Sequences" and solve exercise 2 in page 23.

5. Let (E, B, π) be a Serre fibration, where B is simply connected and the fiber is a homology n-sphere for $n \geq 1$. Show that there is an exact sequence:

   $$... \to H_r(E) \to H_r(B) \to H_{r-n-1}(B) \to H_{r-1}(E) \to ...$$

   In particular, for $0 \leq r \leq n-1$, $H_r(B) \simeq H_r(E)$.