Homework 6

Due: Mar 9

1. Compute the obstruction class of the tangent bundle of S^2, $o_2(\tau_{S^2})$, directly from its definition.
2. Let $\xi = (E, B, \pi)$ be an oriented k-dimensional vector bundle over a CW complex B. Prove that there exists a nowhere vanishing section for ξ, defined over the k-skeleton B^k iff $e(\xi) = 0 \in H^k(B; \mathbb{Z})$. (Hint: Given a nowhere vanishing section s_{k-1} of ξ defined over B^{k-1} such that $o(s_{k-1}) = \delta\eta$ i.e. $[o(s_{k-1})] = 0$, modify s_{k-1} to a new section t_{k-1} over the $(k-1)$-skeleton, so that over each $(k-1)$-cell, s_{k-1} and t_{k-1} differ by η.)
3. Read the proof of Theorem 11.8 in Hutchings note: "Introduction to higher homotopy groups and obstruction theory " and build a S^1-bundle over E_g, surface of genus g, with Euler class $k \in \mathbb{Z} \cong H^2(E_g; \mathbb{Z})$. What is the total space of such bundle for $g = 0$?
4. (a) Milnor-Stasheff: 12-A

   (b) Compute w_1 of $T^2 \# \mathbb{RP}^2$.
5. Milnor-Stasheff: 12-C